VOLUME LXXIV NUMBER 4

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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NOVEMBER 1931

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

November 1931 Vol. LXXIV, No. 4

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The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶ The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, Canary Islands, El Salvador, Argentina, Bolivia, Brazil, Colombia, Chile, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Hayti, Uruguay, Paraguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Balearic Islands, Spain, and Venezuela. ¶ Postage is charged extra as follows: for Canada and Newfoundland, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies, 5 cents (total 80 cents). ¶ Patrons are requested to make all remittances payable to The University of Chicago Press, in postal or express money orders or bank drafts.

The following are authorised agents:

For the British Empire, except North America, India, and Australasia: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 9s. 3d. each; single copies, including postage, 3s. 9d. each.

For Japan: The Maruzen Company, Ltd., Tokyo.

For China: The Commercial Press, Ltd., Paoshan Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "Observatory, Williamsbay, Wisconsin."

The articles in this journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Applications for permission to quote from this journal should be addressed to The University of Chicago Press, and will be freely granted.

Entered as second-class matter, January 17, 1895, at the Post-Office at Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 18, 1918.

PRINTED IN THE U.S.A.

THE ASTROPHYSICAL JOURNAL

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VOLUME LXXIV NOVEMBER 1931 NUMBER 4

A STUDY OF THE SPECTRA OF B STARS

By OTTO STRUVE

ABSTRACT

Thirty-five spectrograms of thirteen stars, ranging in spectral type from O9 to B8, were measured for the purpose of obtaining a reasonably complete *list of absorption lines* in the hottest stars. The list contains 379 lines (between λ 3820 and λ 4924), of which about 150 are new. Approximately 130 lines have been newly identified. The following elements are present: *H*, *He* I, *He* II, *C* II, *C* III, *N* II, *N* III, *O* II, *O* III, (*Na* I), *Mg* II, *Al* II, *Al* III, *Si* II, *Si* III, *Si* IV, *P* II, *P* III, *S* II, *S* III, *Ca* II, *Ti* II, *Cr* II, *Fe* II, *Ni* II; and the following are suspected: *K* II, *Ca* III, *S* IV.

The *contours of the He* I *lines* were investigated. Members of the series $2P-mD$ and $2p^3-md^3$ were found to be highly susceptible to broadening by *electric fields of ions and electrons* in the reversing layers (Stark effect). Accurate wave-lengths of twenty *He* I lines are given. The wave-lengths of two forbidden lines were found to agree well with laboratory determinations.

The *intensities of several pairs of lines of He* I were estimated in 116 stars of types O to B9. The ratio of intensity singlet/triplet is greatest for stars of type B2 and decreases rapidly toward class O and slowly toward B9. This peculiar behavior of the singlets is not due to temperature-ionization, nor to effects of pressure or of electrical fields. It is possible that the effect is caused by a change of the atomic absorption coefficient depending upon outside conditions, or by a departure from thermo-dynamic equilibrium.

The *members of the Fowler series of He* II were found to be *broadened by Stark effect*, in agreement with theory.

Several lines of *N* II and of *O* II which are diffuse in the laboratory (except under very low pressures) show no evidence of broadening in stellar spectra.

The members of doublets and of quartets in *O* II show approximately the same relative intensities in the stars as they do in the laboratory.

The *lines of silicon were used for a test of Milne's generalized ionization equations.* It is shown that lines belonging to two stages of ionization of silicon are simultaneously strengthened in giants. The earlier form of the ionization theory fails to explain this phenomenon. It is probably a consequence of Milne's so-called "null-effect."

An attempt was made to estimate an *upper value for possible deviations in the abundance of silicon in individual stars.* The ratio does not exceed 5:1, and probably remains below 2:1.

It is the purpose of this investigation to provide a foundation for further work on the spectra of early-type stars. A preliminary examination of spectrograms of several B-type stars showed that there

were many absorption lines which did not appear in any of the published lists of stellar wave-lengths.[1] I therefore decided to measure a number of spectrograms in order to have a reasonably complete list of absorption lines present in classes O and B.

The observational material consisted of twelve plates taken with the Coudé spectrograph of the Mount Wilson Observatory, and of twenty-three plates taken with the Bruce spectrograph of the Yerkes Observatory. For nearly all of the spectrograms I used Eastman Process plates, which give excellent contrast and bring out many lines which are invisible on fast photographic emulsions such as Eastman-40.

Table I gives a list of thirteen stars selected for measurement. It was my intention to cover, as nearly as possible, all subdivisions of the Harvard classification between O*e*5 (or O9) and B8. The selection was based almost exclusively upon the sharpness of the lines. Not many years ago such a selection would have been regarded as not sufficiently representative of the whole spectral class, because there are many stars which have wide and diffuse lines. It is now regarded as almost certain that the dish-shaped contours of these wide and diffuse lines are produced by rapid axial rotation—a factor quite extraneous to the structure of the reversing layer. It is therefore safe to say that our list is representative for normal spectra of class B. It includes a number of stars of high luminosity, such as 67 Ophiuchi and β Orionis, and others of comparatively low luminosity, such as γ Pegasi and ι Herculis.

The plates were measured in the usual way and reduced by means of Hartmann formulae. I found it expedient to measure the wave-lengths of the faintest lines in one direction only. If all lines had been of the same intensity, this procedure would have involved no systematic errors of any sort, because the personal difference in setting upon stellar absorption lines and upon emission lines of the comparison spectrum is eliminated in applying the correction for radial velocity. But the lines are not all of the same intensity. There-

[1] F. E. Baxandall, *Comparison of the Spectra of Rigelian, Crucian and Alnitamian Stars* (Solar Physics Committee), 1914; H. H. Plaskett, *Publications of the Dominion Astrophysical Observatory* (Victoria), **1**, 351, 1922; F. Henroteau and J. P. Henderson, *Publications of the Dominion Observatory* (Ottawa), **5**, 5, 1920; B. H. Dawson, *Publications of the Astronomical Observatory of the University of Michigan*, **2**, 159, 1916; etc.

fore, the possibility remains that faint lines are affected by an error of setting: the procedure is strictly correct only for lines of the same

TABLE I

OBSERVATIONAL MATERIAL

Star	Sp. H. D.	Plate	Date	U.T.	Instrument	Linear Disp. at λ4500	Measured Range (λλ)	No. of Measures
10 Lac.....	O9	1R 9491	1930 Aug. 23	3^h45^m	Y. O. 40″	30 A/mm	3958–4713	1
		1R 9506	Sept. 4	8 40	Y. O. 40″	30	3958–4713	1
15 κ Cas...	Bo	1B 1727*	1908 Sept. 8	15 10	Y. O. 40″	30	3999–4572	1
		1B 2482*	1910 Aug. 12	19 46	Y. O. 40″	30	4572–4982	1
37 ϕ^1 Ori...	Bo	1R 9661	1931 Feb. 25	1 50	Y. O. 40″	30	3990–4172	1
23 τ Sco...	Bo	1R 9709	Apr. 11	9 21	Y. O. 40″	30	4012–4713	1
2 β CMa..	B1	R 1771	1930 Mar. 3	1 21	Y. O. 40″	10	4338–4759	1
		1R 9307	Mar. 6	0 40	Y. O. 40″	30	3901–4717	1
8 β Cep...	B1	1R 9548	Oct. 19	2 12	Y. O. 40″	30	3932–4713	1
88 γ Peg...	B2	B 246*	1901 Nov. 27	12 11	Y. O. 40″	10	4172–4885	1
		1R 9249	1929 July 14	7 00	Y. O. 40″	30	3913–4164	1
		1R 9253	July 17	7 08	Y. O. 40″	30	3761–4274	2
		R 1588	Sept. 7	4 08	Y. O. 40″	10	4338–4713	1†
		R 1599	Sept. 21	4 38	Y. O. 40″	10	4338–4722	2
		C 342	1930 Oct. 1	9 09	Mt. W. 100″	12	3840–4603	1
		C 345	Oct. 2	7 45	Mt. W. 100″	12	3840–4716	2
82 δ Cet...	B2	1B 4323*	1915 Dec. 14	13 16	Y. O. 40″	30	3958–4555	1
		1R 9513	1930 Sept. 9	8 31	Y. O. 40″	30	3958–4713	1
24 γ Ori....	B2	C 63	1927 Jan. 23	8 00	Mt. W. 100″	3	4338–4495	1
		C 217	1929 Oct. 23	11 03	Mt. W. 100″	6	3957–4603	2
17 ζ Cas...	B3	1R 9269	July 29	8 11	Y. O. 40″	30	3901–4384	1**
85 ι Her....	B3	1R 9242	July 3	7 11	Y. O. 40″	30	3913–4172	2‡
		C 313	1930 June 18	7 55	Mt. W. 100″	12	3930–4495	1
		C 314	June 19	5 27	Mt. W. 100″	12	3930–4495	1
		C 325	July 5	7 40	Mt. W. 100″	12	3903–4713	4§
67 Oph....	B5p	1R 9256	1929 July 19	5 54	Y. O. 40″	30	3913–4481	1
		1R 9262	July 27	4 11	Y. O. 40″	30	3913–4481	2‡
		C 315	1930 June 19	9 43	Mt. W. 100″	12	3930–4495	1
		C 326	July 6	6 53	Mt. W. 100″	12	3903–4727	4
19 β Ori....	B8p	C 122	1928 Dec. 12	7 33	Mt. W. 100″	6	4005–4737	1
		C 129	Dec. 30	7 50	Mt. W. 100″	6	4326–4957	1
		C 161	1929 Mar. 26	3 17	Mt. W. 100″	6	4326–4957	1
		R 1768	1930 Feb. 20	0 06	Y. O. 40″	10	4338–4759	1
		1R 9515	Sept. 9	10 40	Y. O. 40″	30	3913–4717	1
		1R 9516	Sept. 9	10 54	Y. O. 40″	30	3958–4716	1

* The time is G.M.T. counted from noon.

† Measured by F. E. Roach.

‡ One measure by P. C. Keenan.

§ One measure by F. E. Roach.

** Only a few of the stronger lines measured on this plate.

intensity as those from which the correction for radial velocity was derived. But in practice this does not appreciably affect the results. The probable error of setting on a faint line so much exceeds any

systematic error that the latter may be disregarded. The wave-lengths of the stronger lines were determined in the usual way from measurements made in two directions, and should be accurate within a few hundredths of an Ångstrom unit.

The wave-lengths of the lines and their estimated intensities appear in Table II. Intensity 1 denotes a very faint line which can be seen only with difficulty. A faint, but easily visible, line is denoted by 3. The strongest lines, other than those of hydrogen, have an intensity of approximately 10, and the hydrogen lines are estimated at about 20. Considerable attention has been devoted to the faintest lines: those of intensity 1 were carefully scrutinized and rejected from the table unless they appeared on several plates or unless they were definitely identified with elements already known to be present.

The work was confined to the ordinary photographic region of the spectrum, which was further limited on the violet side by the transmission curve of the spectrograph. The list of wave-lengths is reasonably complete between λ 3950 and λ 4700. The lack of sensitivity of the Eastman Process emulsion to radiations of wave-lengths longer than λ 4700 made it necessary to employ a coarser emulsion for the region between λ 4700 and λ 4900; and consequently only the stronger lines could be measured. In the case of γ Pegasi the measurements extend on the violet side to λ 3850.

Of the thirteen stars in Table I, ten are separately listed in Table II. The remaining three stars—ζ Cassiopeiae, φ^1 Orionis, and γ Orionis—were used in determining the mean wave-lengths but are not given in the section entitled "Intensities." φ^1 Orionis was measured in detail over a narrow range of wave-lengths, and in ζ Cassiopeiae and γ Orionis no attempt was made to measure the faintest lines.

The wave-lengths in Table II are the arithmetical means of all independent determinations. Unit weight was assigned to each measurement, irrespective of the intensity of the line. This procedure was adopted to avoid throwing too much weight upon any one measurement. Some of the best plates available, principally those from Mount Wilson, were measured several times. Since each measurement was given weight 1, this somewhat increases the effect of the better plates upon the adopted wave-lengths.

TABLE II

WAVE-LENGTHS OF LINES IN B-TYPE STARS

OBSERVED WAVE-LENGTH I.A.	INTENSITY										IDENTIFICATION
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
3820	...	...	...	...	...	15	...	...	...	...	*He* I 9.61(4); *He* I 9.75(1)
3835	...	...	...	...	...	20	...	...	...	...	*Hη* 5.39
3850.94	...	...	...	...	...	1	...	...	...	...	*O* II 1.04(3); *O* II 0.81(2); *O* II 1.58(2)
3856.01	...	...	...	...	...	2	...	...	...	...	*O* II 6.16(5); *N* II 6.07(3); *Si* II 6.02(8)
3857.10	...	...	...	...	...	1	...	...	...	...	*O* II 7.18(4)
3860.54	...	...	...	...	...	1	...	...	...	...	.....
3862.53	...	...	...	...	...	2	...	...	...	...	*Si* II 2.59(6)
3864.45	...	...	...	...	...	1	...	...	...	...	*O* II 4.45(5); *O* II 4.68(1)
3867.55	...	...	...	...	...	9	...	...	...	...	*He* I 7.46(2); *He* I 7.62(1)
3869.88	...	...	...	...	...	1	...	...	...	...	[*Al* II 0.06(½)]
3871.88	...	...	...	...	...	2	...	...	...	...	*He* I 1.80(1)
3874.25	...	...	...	...	...	1	...	...	...	...	*O* II 4.10(2)
3876.27	...	...	...	...	...	3	...	...	...	...	*C* II 6.19(4); *O* II 5.82(4); *C* II 6.41(2); *C* II 6.67(1); *C* II 6.05(1)
3878.43	...	...	...	...	...	1	...	...	...	...	*He* I 8.18(1)
3882.34	...	...	...	...	...	1	...	...	...	...	*O* II 2.19(7); *O* II 2.45(1)
3889.02	...	...	...	...	...	20	...	...	...	...	*Hζ* 9.05; *He* I 8.65(10); *C* II 9.18(1)
3899.53	...	...	...	...	...	1	...	...	...	...	[*Al* II 0.68(10)]
3907.49	...	...	...	2	...	1	...	...	...	...	*O* II 7.45(4)
3911.99	...	...	...	7	...	3	...	...	...	...	*O* II 1.95(10); *O* II 2.09(2)
3918.95	...	...	...	6	...	4	...	4	4	...	*C* II 8.98(6); *O* II 9.28(6); *N* II 9.00(6)
3920.65	...	...	...	5	...	4	...	5	4	...	*C* II 0.68(8)
3923.30	...	...	...	1	...	2	...	1	1	...	*S* II 3.47(4)
3924.42	...	...	...	4	...	2	...	...	...	...	*Si* III 4.44(4)
3926.59	...	...	...	4	...	3	...	3	3	...	*He* I 6.53(1)
3928.59	...	...	...	2	...	1	...	...	(1)	...	*S* III 8.59(5)
3933.14	...	...	...	...	...	1	...	...	...	...	*S* II 3.28(5)
3933.60	...	...	...	3	2	4	...	8	6	10	*Ca* II 3.66(10)
3936.05	...	...	...	2	1	1	...	1	2	1	*He* I 5.91(1)
3938.20	...	...	...	...	...	1	...	...	1	...	[*Al* II 8.62(½)]
3939.69	...	...	...	1	1	(1)	...	...	...	...	.....
3945.01	...	...	...	4	2	2	...	...	...	...	*O* II 5.05(5)
3954.30	...	...	...	4	3	2	...	...	...	...	*O* II 4.37(7)
3955.80	...	...	...	2	...	2	...	...	...	...	*N* II 5.85(6)
3961.52	3	...	...	...	...	...	...	...	...	...	*O* III 1.59(8); *S* III 1.56(3)
3964.67	2	...	...	8	5	6	4	6	7	4	*He* I 4.73(4)
3966.91	...	...	...	...	...	...	...	...	1	...	[*O* II 7.44(1)]
3968.46	2	...	...	...	...	4	...	6	7	9	*Ca* II 8.46(10)
3970.16	20	...	...	30	40	23	...	15	22	20	*Hε* 0.08; *S* II 0.72(3)
3973.28	...	...	...	5	4	2	...	...	...	1	*O* II 3.27(10)
3978.30	...	...	...	...	...	1	...	...	...	...	.....

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
3979.67	...	...	...	...	...	...	...	...	...	1	*S* II 9.85(2)
3980.24	...	...	...	...	...	1	...	...	...	...	*Al* III 0.56(2); *C* II 0.35(2)
3982.62	1	...	...	3	2	2	1	...	...	...	*O* II 2.72(5)
3983.73	...	...	...	1	1	1	...	...	...	...	*S* III 3.75(4); [*Al* II 3.7(½)]
3985.80	...	...	...	2	...	1	...	...	...	...	*S* III 5.95(4)
3989.35	...	...	...	...	...	1	...	...	...	1	.....
3991.78	...	...	...	...	...	1	...	...	...	...	*Si* II 1.77(2)
3993.33	...	...	...	1	...	1	...	...	2	2	*S* II 3.49(6)
3994.98	2	...	...	4	4	4	3	2	3	3	*N* II 5.00(10); [*Al* II 5.86(5)]
3997.64	...	...	...	...	...	1	...	...	...	...	*Si* II 8.00(1); *S* II 8.77(3)
4003.37	2	...	...	...	...	...	...	...	...	...	*N* III 3.64(4)
4005.03	...	...	...	...	...	1	...	...	...	...	.....
4007.53	...	...	...	(1)	...	1	...	1	...	...	Forbidden *He* I 7.79
4009.28	2	6	...	6	6	9	8	5	8	3	*He* I 9.27(1); [*C* II 9.90(2)]
4015.41	1	...	...	...	1	2	1	1	2	1	*Ni* II 5.50(1)
4024.05	1	...	...	...	1	2	2	1	1	...	*He* I 3.99(1); *O* II 4.04(1)
4025.32	...	...	...	...	1	1	2	3	...	...	Forbidden *He* I 5.48
4026.21	10	8	10	15	12	12	15	15	11	8	*He* I 6.19(5); *He* I 6.36(1); *Al* II 6.5(5); *N* II 6.09(3)
4028.73	...	...	...	...	...	...	1	...	1	2	*S* II 8.80(5)
4030.51	...	...	...	...	...	1	...	...	1	1	*Al* II 0.87(0); *Al* II 1.14(½)
4032.90	...	...	...	...	...	1	...	...	1	1	*S* II 2.80(5)
4035.12	...	...	2	2	2	2	3	1	1	...	*N* II 5.09(4); *O* II 5.00(0)
4039.24	...	...	(1)	...	1	1	1	...	...	...	*Al* II 9.30(½); *Al* II 9.40 (0)
4041.28	...	...	1	3	3	2	3	1	2	1	*N* II 1.32(5); *O* II 1.20(0)
4043.46	...	...	...	2	1	2	2	1	1	1	*N* II 3.54(3)
4045.00	...	...	...	...	...	1	...	1	...	...	.....
4046.88	...	...	...	...	...	1	...	...	1	...	.....
4048.15	...	1	...	2	...	1	...	...	...	...	*O* II 8.22(1)
4053.08	...	...	...	...	...	1	...	...	...	...	.....
4054.09	2	...	...	...	1	...	...	...	...	...	.....
4056.01	2	...	...	...	...	...	...	...	...	...	.....
4057.09	...	...	(1)	...	...	1	1	...	1	1	*P* III 7.39(4); *N* II 7.00(1)
4059.24	1	...	...	...	...	1	...	...	...	...	*P* III 9.27(6)
4060.74	1	...	1	2	2	1	...	...	...	...	*O* II 0.58(3); *O* II 0.98(2)
4062.90	...	...	...	2	1	1	...	...	...	...	*O* II 2.90(1)
4064.89	...	...	1	...	...	1	...	...	...	...	*S* III 4.40(3)
4067.04	...	...	...	...	...	1	...	...	...	1	*Ni* II 7.04(3)
4067.88	2	...	...	1	...	...	...	...	...	...	.....
4069.11	2	...	...	...	...	...	...	...	...	...	[*C* II 8.97(3)]
4069.81	4	3	3	7	5	4	4	2	2	1	*O* II 9.90(6); *O* II 9.64(4); *C* II 0.30(3); *C* II 8.97(3); *S* II 9.65(1); *S* II 9.89(1)
4072.11	2	2	2	5	4	4	3	1	2	1	*O* II 2.16(8); *P* II 2.13(5)
4073.32	...	...	...	...	...	1	2	...	...	...	*N* II 3.04(2)
4074.00	2	...	...	...	...	...	...	...	...	...	*O* III 3.90(0)
4074.53	...	...	1	2	2	2	2	1	1	1	*C* II 4.53(2); *C* II 4.89(1)
4075.88	4	4	3	6	4	3	3	1	1	...	*O* II 5.87(10)

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
4077.45	...	...	...	1	1	1	...	...	...	...	.....
4078.86	...	2	1	3	3	2	2	...	...	...	*O* II 8.86(4); *O* II 9.00(0)
4080.07	...	...	...	...	...	1	...	...	...	...	*P* III 0.04(7)
4081.01	1	...	...	2	1	1	1	...	...	...	*O* III 1.10(1)
4082.01	...	1	...	...	1	1	...	...	...	1	*N* II 2.28(2)
4083.87	...	...	...	2	2	1	2	...	...	...	*O* II 3.90(2)
4084.44	1	...	...	...	...	...	...	...	...	...	.....
4085.01	...	2	1	3	3	2	...	...	...	...	*O* II 5.12(3)
4087.16	...	...	...	3	2	2	1	...	...	...	*O* II 7.16(2); *N* II 7.35(0)
4088.95	8	...	7	...	...	...	...	...	...	...	*Si* IV 8.86(10)
4089.19	...	6	...	6	5	2	1	...	...	...	*O* II 9.28(4); *Si* IV 8.86(10)
4091.44	...	...	1	...	...	1	...	...	...	...	*P* II 1.53(5)
4093.03	1	2	2	2	1	1	1	...	1	1	*O* II 2.94(5); [*N* II 2.65(0)]
4095.45	...	1	1	...	...	...	...	...	...	...	[*O* II 5.63(0)]
4097.26	6	3	2	3	2	...	...	...	...	...	*O* II 7.25(4); *N* III 7.31(10); *O* II 7.32(1)
4099.58	...	...	...	...	...	...	...	...	1	...	*S* II 9.42(3); *S* II 9.27(2)
4100.19	1	...	...	1	...	...	...	...	...	...	*He* II 0.08
4101.76	20	8	30	20	20	30	50	50	30	20	*Hδ* 1.74
4103.37	3	...	...	...	...	...	...	...	...	...	*N* III 3.37(9)
4104.92	...	1	...	4	3	...	...	...	...	1	*O* II 5.00(7); *O* II 4.74(5)
4107.40	...	1	1	1	...	...	...	...	1	...	*O* II 7.07(1)
4110.56	...	...	...	3	1	1	...	...	1	1	*O* II 0.80(3); *O* II 0.20(1)
4112.01	...	...	...	2	1	1	...	...	...	...	*O* II 2.04(4)
4116.09	6	3	7	3	3	1	...	...	...	...	*Si* IV 6.10(8)
4119.26	2	3	3	4	4	3	3	...	...	...	*O* II 9.22(8)
4120.84	4	×	5	6	6	8	8	8	6	3	*He* I 0.81(3); *He* I 0.98(1); *O* II 0.30(3); *O* II 1.48(4); *O* II 0.55(2)
4122.75	...	...	...	...	...	1	...	...	...	1	.....
4124.56	...	...	...	...	...	...	...	...	...	1	.....
4128.04	...	...	...	...	...	1	1	4	5	6	*Si* II 8.05(8); *P* II 7.49(6)
4129.02	...	1	2	1	1	1	...	1	...	...	*O* II 9.34(2)
4129.58	...	...	...	...	...	...	...	...	...	1	.....
4130.88	...	...	(1)	...	...	1	2	4	5	6	*Si* II 0.88(10); *S* II 0.90(2)
4131.87	...	...	...	...	...	1	...	...	...	...	.....
4132.89	...	...	2	3	3	2	2	1	1	1	*O* II 2.82(6)
4135.10	...	...	...	...	...	...	2	...	...	...	.....
4137.72	...	(1)	...	...	...	2	...	...	...	...	.....
4139.79	...	...	...	...	...	1	...	...	...	...	*Al* III 0.21(2)
4140.82	...	...	...	...	...	1	...	...	...	...	[*O* II 0.74(0)]
4142.22	...	...	...	...	...	...	...	...	2	1	*S* II 2.31(4)
4143.80	5	(3)	5	8	8	11	9	10	8	4	*He* I 3.77(2); *O* II 3.77(2); *O* II 3.52(1); forbidden *He* I 3.55;
4145.08	...	...	...	...	...	...	...	...	2	...	*S* II 5.16(4)
4146.00	...	...	...	...	...	...	1	...	...	...	*O* II 6.09(3); *N* II 5.76(3); *O* II 5.90(0)

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
4150.08	...	...	...	1	2	2	2	2	1	...	*Al* III 9.90(3); *Al* III 0.14(2); *Al* III 9.92(1)
4150.27	...	...	...	...	...	...	...	...	...	1	.....
4153.15	2	2	2	4	3	2	3	2	3	2	*O* II 3.31(7); *S* II 3.14(6)
4155.09	...	...	...	...	...	1	...	...	...	...	[*N* II 5.0(0)]
4156.56	2	...	...	2	1	1	...	...	1	...	*O* II 6.54(3); [*N* II 6.8(0)]
4159.45	...	...	...	...	...	1	...	...	1	...	*Al* II 0.26(3); *Al* II 0.24(2); *Al* II 9.45(1)
4162.61	...	...	...	2	...	2	1	1	3	2	*S* II 2.73(6); *S* II 2.37(3)
4163.00	2	...	1	...	...	...	...	...	...	...	.....
4164.78	...	...	...	3	3	3	3	1	2	...	[*S* II 4.98(2)]
4166.90	...	...	...	1	1	1	1	...	2	...	.....
4169.04	2	×	2	4	3	5	4	4	2	2	*He* I 8.97(1); *O* II 9.23(4); *Al* II 8.51(1); *Al* II 8.42($\frac{1}{2}$)
4171.62	...	...	...	1	1	1	1	...	...	...	*N* II 1.63(2)
4173.59	...	...	...	...	...	(1)	...	...	...	2	*Fe* II 3.48(6); *S* II 4.33(5); *S* II 4.07(4)
4174.12	...	...	...	...	1	1	1	2	1	...	*S* II 4.33(5); *S* II 4.07(4)
4176.23	...	...	1	1	1	1	2	...	...	...	*N* II 6.17(3)
4178.91	...	...	...	...	...	...	...	...	1	2	*Fe* II 8.87(6)
4179.50	...	...	...	2	...	1	1	...	...	...	*N* II 9.68(1)
4185.46	...	2	2	3	2	2	...	...	...	...	*O* II 5.45(8)
4186.83	3	...	2	...	...	1	...	...	...	...	.....
4187.55	...	...	...	1	1	1	...	...	...	...	.....
4188.74	...	...	...	...	...	1	...	...	...	...	*Al* III 8.88(5)
4189.74	2	(3)	2	4	3	2	2	1	1	...	*O* II 9.79(10); *S* II 0.07(2)
4192.57	1	...	...	2	...	...	...	...	...	...	.....
4195.70	2	...	1	1	...	...	...	...	...	...	*N* III 5.70(5)
4199.84	...	...	...	...	...	1	...	...	1	...	*Al* III 9.00(5)
4199.92	2	(2)	2	...	...	...	...	...	...	...	*He* II 9.87; *N* III 0.02(6)
4202.89	...	...	(1)	...	...	...	...	...	1	1	.....
4207.15	...	...	...	1	...	1	...	...	...	...	*N* II 7.51(1); *N* II 6.57(0); *N* II 6.35(0)
4211.36	...	...	...	...	...	1	...	...	...	...	.....
4212.41	3	...	...	...	...	...	...	...	...	...	*Si* IV 2.44(3)
4220.07	1	...	...	...	...	...	1	...	...	...	.....
4222.36	...	...	...	1	...	2	1	...	...	...	*P* III 2.15(7)
4227.83	...	...	...	2	1	1	2	...	...	...	*N* II 7.83(3); *Al* II 6.81(6); *Al* II 7.49(5); *Al* II 7.98(4)
4230.76	...	...	...	...	...	1	...	...	1	...	*S* II 0.98(3)
4232.04	...	...	...	2	...	...	...	...	...	...	.....
4233.10	...	...	...	...	...	...	...	1	3	4	*Fe* II 3.16(8)
4237.01	...	...	1	2	2	2	2	1	1	2	*N* II 6.98(6)
4238.83	...	...	...	...	...	1	...	...	...	1	.....
4241.76	...	...	2	3	2	2	3	1	2	(1)	*N* II 1.80(8)
4243.84	...	...	...	...	...	1	...	...	...	...	.....
4246.50	1	...	...	...	...	1	...	...	...	...	*P* III 6.68(7)
4248.95	...	...	...	1	...	1	...	...	...	...	.....

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
4250.10	1	...	...	...	...	...	...	...	...	...	.....
4253.66	2	4	4	5	5	3	...	1	1	...	*O* II 3.98(8); *S* III 3.51(6)
4263.81	...	...	...	1	...	...	1	...	...	...	.....
4267.16	2	2	4	5	5	9	5	10	9	4	*C* II 7.27(10); *C* II 7.02(8); *S* II 7.86(5); *S* II 7.27(4); *S* II 6.90(4)
4269.43	...	...	...	...	...	...	...	...	...	1	*S* II 9.80(2)
4271.78	...	...	...	...	...	1	...	...	...	1	.....
4273.35	...	...	1	...	1	1	...	...	...	1	[*O* II 3.17(0)]
4275.68	2	...	3	3	3	2	2	...	(1)	...	*O* II 5.52(4)
4276.95	...	...	1	...	...	...	...	...	...	...	*O* II 6.71(1)
4277.55	...	2	2	2	1	...	(1)	...	...	...	*O* II 7.40(1); *O* II 7.90(1)
4278.60	...	...	...	...	...	(1)	...	1	1	...	*S* II 8.62(3)
4281.22	...	...	...	1	1	1	...	...	...	...	*O* II 1.40(0)
4283.04	...	...	...	2	1	1	...	2	2	...	*O* II 2.96(1); *S* II 2.65(3); *O* II 3.75(0); *Al* II 2.97($\frac{1}{2}$)
4285.14	...	...	2	3	2	1	...	...	...	...	*S* III 5.00(5); *O* II 5.70(3)
4286.24	...	...	...	...	...	1	...	...	...	...	.....
4288.93	...	...	1	1	...	...	...	...	...	...	*O* II 8.83(1)
4291.48	...	...	1	2	1	...	...	...	...	...	*O* II 1.25(1); *O* II 2.23(0)
4294.27	...	...	...	...	...	...	...	3	2	2	*S* II 4.44(6)
4294.87	1	...	2	2	2	(2)	...	...	...	...	*O* II 4.82(3)
4296.54	...	...	1	...	...	1	...	...	...	2	*Fe* II 6.56(6)
4298.94	...	...	...	...	...	1	...	...	...	...	.....
4299.88	...	...	1	...	...	...	...	...	1	1	[*Ti* II 0.05(12)]
4303.28	...	...	...	...	...	...	...	...	1	2	*Fe* II 3.18(4); *S* II 3.78(1)
4303.90	2	...	2	3	2	1	...	...	...	...	*O* II 3.82(5)
4307.37	...	...	...	2	1	...	...	...	...	...	*O* II 7.31(1); [*Al* II 7.20(3)]
4308.42	...	...	...	...	...	...	...	...	...	1	*Ti* II 7.89(12)
4310.54	...	...	...	...	...	1	...	...	...	...	.....
4312.22	...	...	...	...	...	1	...	...	...	...	*O* II 2.10(0)
4313.33	...	...	1	...	...	1	...	...	...	...	*O* II 3.43(1)
4314.44	...	...	...	...	...	...	...	...	...	1	[*Ti* II 4.98(5)]
4315.27	...	...	1	...	...	1	1	...	...	...	*O* II 5.55(0)
4317.21	...	3	3	4	3	3	2	3	1	...	*O* II 7.16(8)
4318.45	...	...	...	...	...	1	...	...	...	...	*S* II 8.66(3)
4319.66	1	3	4	5	3	3	2	...	...	...	*O* II 9.65(8); *O* II 9.93(1)
4325.64	2	...	3	3	2	1	1	...	...	(2)	*O* II 5.77(3)
4327.75	...	...	1	...	1	...	...	...	...	...	*O* II 7.48(3); *O* II 7.89(0)
4331.76	...	...	...	2	...	1	...	...	...	...	*O* II 1.89(2); *O* II 1.47(0); *O* II 1.21(0)
4332.68	...	1	1	...	...	...	...	...	...	...	*S* III 2.69(6); *O* II 2.76(1)
4336.91	...	...	...	3	2	1	...	...	...	...	*O* II 6.86(6)
4337.85	...	...	...	...	...	...	...	...	1	1	*Ti* II 7.92(10)
4340.47	20	10	20	20	15	40	40	50	24	20	*Hγ* 0.47; *O* II 0.36(2); *S* III 0.25(6)
4342.90	...	1	1	...	1	...	...	...	...	...	*O* II 2.00(4); *O* II 3.30(0)
4345.56	...	...	2	3	3	2	2	...	...	...	*O* II 5.57(7)

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
4347.47	1	...	2	2	3	2	1	...	1	...	*O* II 7.43(5); *Al* II 7.78(4); *Al* II 7.80(3)
4348.21	...	...	...	...	...	1	...	...	...	1	.....
4349.45	3	4	3	4	4	3	2	1	2	1	*O* II 9.44(8)
4350.25	...	...	...	...	...	1	...	...	...	...	.....
4351.27	2	...	3	3	3	3	2	...	(2)	...	*O* II 1.28(6)
4351.79	...	...	...	...	...	...	...	...	...	2	*Fe* II 1.77(6)
4352.47	(1)	...	...	...	...	1	...	...	...	...	.....
4353.84	...	2	...	...	...	...	1	...	...	...	*O* II 3.60(1)
4354.60	...	...	1	...	...	1	...	...	...	1	*S* III 4.58(5)
4358.06	...	...	...	1	...	...	...	...	...	(1)	*O* II 7.25(0); *O* II 8.40(0)
4361.51	2	...	1	2	1	1	1	...	...	1	*S* III 1.57(6)
4364.91	...	...	...	1	...	1	..	...	...	...	*S* III 4.77(4); *Al* III 4.59(2)
4366.88	2	5	3	4	3	3	2	...	...	...	*O* II 6.91(7)
4369.30	...	...	...	1	...	2	...	...	...	...	*O* II 9.28(4)
4370.31	...	...	(1)	...	...	1	1	...	...	...	.....
4372.30	...	1	...	3	2	2	3	...	...	...	*O* II 1.65(2); *C* II 2.49(1); *C* II 1.59(0)
4374.36	...	...	...	...	...	2	1	...	...	...	*C* II 4.28(2)
4378.36	...	1	...	2	1	1	...	...	...	...	*O* II 8.40(3)
4379.22	4	...	2	...	...	...	...	...	...	...	*N* III 9.09(10); *O* III 9.55(3)
4382.43	2	...	...	...	...	2	...	...	...	...	.....
4383.22	...	...	...	...	...	1	...	1	...	...	[Forbidden *He* I 3.26]
4384.86	1	...	...	...	...	...	...	...	...	...	.....
4385.28	...	...	...	...	...	...	...	...	...	2	*Fe* II 5.39
4387.93	9	6	8	10	10	17	10	15	9	4	*He* I 7.93(3)
4390.55	...	...	...	...	...	2	...	2	1	2	*Mg* II 0.58(10)
4391.96	2	...	...	1	...	2	...	...	...	...	[*S* II 1.94(1)]
4395.88	...	...	1	3	2	2	2	2	(1)	(2)	*O* II 5.95(7)
4400.40	...	...	...	...	2	(1)	...	...	...	...	*Ca* III 9.64(10)
4404.26	...	...	...	...	...	2	...	...	...	...	.....
4405.13	...	...	1	1	...	...	...	...	...	...	.....
4405.99	...	...	...	...	...	1	...	...	...	...	*O* II 6.02(1)
4408.23	...	...	...	...	...	1	...	...	...	...	.....
4409.32	...	...	...	1	1	...	...	...	...	...	*C* II 1.20(2); *C* II 1.52(2); *C* II 0.06(1)
4411.43	...	...	1	1	1	3	1	1	1	(1)	.....
4414.90	...	2	4	5	4	5	3	2	1	...	*O* II 4.89(10); [*P* II 4.28(6)]
4416.74	...	...	...	...	...	(1)	...	...	...	2	*Fe* II 6.81(4)
4416.97	...	2	3	4	3	4	2	1	2	(2)	*O* II 6.97(8); [*P* II 7.30(5)]
4418.58	...	...	...	...	...	1	...	...	...	...	*S* III 8.82(2)
4419.62	...	...	1	1	1	3	2	1	2	2	.....
4421.05	...	...	...	...	...	1	...	...	...	...	.....
4425.95	...	...	...	...	...	1	...	...	1	...	[*N* II 6.05(0)]
4427.52	...	...	...	1	1	...	...	...	...	...	*N* II 7.21(2); *N* II 7.97(2)
4431.03	...	...	...	1	...	2	...	...	...	1	[*S* II 1.14(1)]
4432.82	...	...	...	1	1	2	2	...	...	...	*N* II 2.71(6); *S* II 2.45(2); [*Al* II 2.82($\frac{1}{2}$)]

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
4433.54	...	...	...	...	...	1	...	...	...	...	*N* II 3.48(2); *Mg* II 3.99(8)
4437.55	2	2	3	3	2	6	4	5	3	2	*He* I 7.55(1)
4441.36	...	...	...	...	...	1	...	...	1	...	*N* II 1.99(3)
4442.96	...	...	1	2	1	2	...	...	...	...	*O* II 3.05(5)
4447.12	1	...	3	2	2	3	3	...	2	...	*N* II 7.04(10); *O* II 7.08(1); [*Al* II 7.8(3)]
4448.31	...	...	1	2	...	1	1	...	...	...	*O* II 8.21(6)
4452.40	...	...	2	2	2	2	2	...	...	1	*O* II 2.38(6)
4463.76	...	...	...	...	...	...	...	...	1	1	*S* II 3.66(3); *S* II 4.45(3)
4465.57	2	1	2	...	...	1	...	...	...	...	*O* II 5.40(4); [*N* II 5.54(0)]
4466.29	...	...	...	3	1	2	...	...	...	...	*O* II 6.32(2)
4468.44	...	...	...	2	...	1	...	...	...	...	*O* II 7.88(4); *O* II 9.32(3); *S* III 7.76(1)
4469.95	2	...	2	2	1	4	3	2	...	...	Forbidden *He* I 9.97
4471.56	11	10	10	10	8	17	12	12	12	10	*He* I 1.48(6); *He* I 1.69(1)
4475.40	...	...	...	...	...	...	...	...	...	1	*P* II 5.26(8)
4477.80	...	...	...	1	1	2	...	...	...	1	*O* II 7.88(2); *N* II 7.74(2); *S* III 8.44(1)
4479.84	...	...	1	1	...	3	2	1	...	...	*Al* III 9.97(4); *Al* III 9.89(3)
4481.25	2	2	3	4	3	9	4	10	8	10	*Mg* II 1.33; *Mg* II 1.13
4483.24	...	...	...	...	...	2	...	...	...	1	*S* II 3.48(4)
4488.33	...	...	2	2	1	...	...	...	...	...	*O* II 8.09(2); *O* II 9.48(1)
4488.71	...	...	...	...	...	...	...	...	...	1	*Fe* II 9.21(4)
4489.51	1	...	...	...	...	1	...	...	...	...	*O* II 9.48(1); [*Al* II 9.87(½)]
4491.03	...	...	2	1	1	1	...	...	...	...	*O* II 1.25(3); *Al* III 0.90(2)
4491.36	...	...	...	...	...	...	...	...	...	1	*Fe* II 1.41(4)
4501.45	...	...	...	...	...	...	...	...	...	1	*Ti* II 1.27(25)
4506.74	...	...	...	1	1	...	...	...	...	...	*O* II 6.50(2); *N* II 7.58(3)
4507.36	2	...	...	...	...	1	...	...	...	...	*N* II 7.58(3); *O* II 6.50(2)
4508.22	...	...	...	...	...	...	...	...	1	2	*Fe* II 8.29(8)
4508.60	...	...	...	...	...	1	...	...	...	...	.....
4510.84	3	...	2	...	...	1	...	...	...	...	*N* III 0.92(6)
4512.51	...	...	...	(1)	1	2	2	...	2	1	*Al* III 2.54(4)
4514.90	4	...	2	...	...	1	...	...	...	...	*N* III 4.89(7)
4515.46	...	...	...	...	...	...	...	...	1	1	*Fe* II 5.33(6)
4518.06	2	...	1	...	...	1	...	...	...	...	*N* III 8.18(3)
4520.15	...	...	...	...	...	...	...	...	...	2	*Fe* II 0.24(6)
4522.51	...	...	...	...	...	...	...	...	2	2	*Fe* II 2.64(6)
4523.52	2	...	1	1	1	...	...	...	...	...	*N* III 3.60(4)
4524.89	...	...	...	...	...	1	...	2	1	1	*S* II 4.89(6)
4526.47	...	...	...	...	...	1	...	...	...	...	.....
4527.09	...	...	...	...	...	...	...	...	1	1	.....
4529.09	1	...	...	2	1	3	2	2	2	1	*Al* III 9.18(6); *Al* III 8.91(1)
4530.47	...	...	1	1	1	2	1	...	...	...	*N* II 0.37(5); *P* II 0.78(7)
4534.42	...	...	...	...	...	...	...	1	...	1	*Mg* II 4.26(4)
4534.90	2	...	...	...	...	...	...	...	...	...	*N* III 4.57(3); *S* IV 5.69(4)
4537.74	...	...	...	...	...	...	...	...	...	1	.....
4541.66	...	...	...	...	...	...	...	...	1	1	*Fe* II 1.52

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
4541.63	4	1	1	...	1	...	...	...	...	...	*He* II 1.63
4549.44	(1)	...	...	...	...	1	...	1	3	3	*Fe* II 9.48(4); *S* II 9.54(4)
4552.64	2	6	4	7	5	6	5	4	5	2	*Si* III 2.65(9); *N* II 2.50(4); *S* II 2.44(5)
4553.93	...	...	...	...	...	2	...	1	...	...	.....
4555.76	...	...	...	...	...	1	...	...	1	2	*Fe* II 5.90(6); [*P* II 4.81(7)]
4558.58	...	...	...	...	...	(1)	...	...	...	2	*Cr* II 8.66; *P* II 8.04(7)
4558.99	1	...	1	...	...	...	...	...	...	...	.....
4561.28	...	...	...	...	...	...	...	1	...	1	.....
4567.88	2	6	3	6	4	4	4	3	4	2	*Si* III 7.87(7)
4571.84	...	...	...	...	...	1	...	...	...	1	*Ti* II 1.98(15)
4572.12	...	...	...	...	2	...	...	...	...	...	*Ca* III 2.11(8)
4574.71	1	2	2	4	3	4	3	1	3	1	*Si* III 4.78(4)
4582.99	...	...	...	1	...	1	...	...	...	...	.....
4583.81	...	...	...	...	...	...	...	...	1	3	*Fe* II 3.84(8)
4586.18	...	...	...	1	1	1	1	..	1	...	*Al* II 5.82(6)
4588.15	...	...	...	...	...	2	...	...	...	1	*Al* II 8.19(5); *Al* II 8.08($\frac{1}{2}$)
4589.84	...	...	...	...	...	1	...	...	...	...	*Al* II 9.75(4); *Al* II 9.69(1)
4590.97	1	4	3	4	3	3	2	...	...	(1)	*O* II 0.98(9)
4592.27	...	...	...	...	...	2	...	...	...	...	.....
4593.88	...	...	...	1	1	1	...	1	...	...	.....
4596.14	...	3	2	2	2	3	2	...	...	...	*O* II 6.19(8)
4597.80	...	...	...	1	...	1	...	...	...	...	.....
4599.18	...	...	...	...	...	1	1	...	1	...	.....
4601.57	...	...	...	...	...	3	2	...	1	1	*N* II 1.49(8)
4602.00	...	2	...	2	2	...	...	...	...	...	*O* II 2.11(2)
4604.88	...	...	...	1	...	1	...	...	...	...	.....
4605.97	...	...	...	...	...	2	...	...	...	...	.....
4607.22	...	...	...	2	2	3	1	...	2	1	*N* II 7.17(7)
4609.70	2	1	2	2	1	3	...	...	...	...	*O* II 9.42(4); *N* II 9.60(0); *O* II 0.14(3)
4611.86	...	...	...	...	...	1	1	...	...	...	.....
4613.78	...	1	1	2	2	2	...	...	1	...	*N* II 3.88(6)
4619.15	...	...	...	1	1	2	2	...	...	1	*C* II 8.85(2)
4621.30	...	1	...	1	1	2	1	...	2	2	*N* II 1.40(7)
4625.16	...	...	...	...	1	2	...	...	...	1	.....
4629.44	...	...	...	...	...	(1)	...	...	1	2	*Fe* II 9.33(4); *P* II 8.70(5)
4630.55	...	3	(3)	3	(3)	5	3	...	4	(1)	*N* II 0.55(10)
4631.12	3	...	...	...	...	...	...	...	...	...	*Si* IV 1.38(3)
4633.75	...	1	...	1	...	1	...	...	...	...	.....
4634.50	2	...	2	...	1	...	1	...	...	...	*N* III 4.16(8)
4636.53	...	...	...	...	...	2	...	...	...	...	.....
4638.71	1	4	2	4	3	4	2	...	...	...	*O* II 8.86(6)
4640.57	...	...	...	...	...	2	...	...	...	...	*Al* II 0.36(4); *Al* II 0.38(3); *N* III 0.64(10)
4641.81	3	7	3	4	4	4	...	1	...	...	*O* II 1.83(9); *N* III 0.64(10) *N* III 1.90(3)
4643.15	...	...	...	...	...	3	2	1	...	1	*N* II 3.11(8)

TABLE II—*Continued*

Observed Wave-Length I.A.	Intensity										Identification
	10 Lac O9	15 κ Cas Bo	23 τ Sco Bo	2 β CMa B1	8 β Cep B1	88 γ Peg B2	82 δ Cet B2	85 ι Her B3	67 Oph B5p	19 β Ori B8p	
4645.21	(1)	...	...	1	...	2	...	...	...	...	.....
4647.42	5	...	3	2	1	2	...	...	...	1	*C* III 7.41(3); *S* II 7.48(2)
4649.23	...	5	3	5	4	4	3	...	2	1	*O* II 9.15(10)
4650.85	5	...	...	3	...	...	...	...	...	...	*C* III 0.24(2); *C* III 1.46(1)
4650.95	...	...	2	...	3	2	...	...	...	...	*O* II 0.85(6); *Al* II 0.54(2); *Al* II 0.65(1)
4654.41	2	...	3	1	...	(1)	...	...	...	...	*Si* IV 4.14(4)
4658.41	1	...	...	1	...	...	...	...	...	...	.....
4661.65	2	4	3	3	3	4	1	...	...	...	*O* II 1.65(9)
4663.25	...	...	...	...	...	1	...	...	...	1	*Al* II 3.05(10)
4666.12	...	...	1	...	1	...	...	...	...	...	.....
4668.98	...	1	...	1	3	...	...	...	...	...	[*O* II 9.53(0)]
4673.90	...	...	1	2	...	1	...	...	...	...	*O* II 3.75(4)
4676.24	2	2	3	3	2	3	...	...	...	...	*O* II 6.25(8)
4678.28	...	...	...	...	...	1	...	...	...	...	*P* II 8.95(6)
4683.76	...	...	...	...	...	1	...	...	...	...	*Si* III 3.77(2)
4685.88	8	(1)	5	1	...	...	...	...	...	...	*He* II 5.81
4691.32	...	...	...	1	...	2	...	...	...	...	*O* II 1.47(1); *O* II 0.97(0)
4694.67	...	...	...	...	...	1	...	...	...	...	*N* II 4.55(3)
4699.08	1	...	2	2	1	3	1	...	...	...	*O* II 9.21(7)
4701.31	...	...	...	...	...	2	...	...	...	...	*O* II 1.23(2); *Al* III 1.65(6); *O* II 1.76(0)
4702.92	...	...	1	2	...	1	...	...	...	...	*O* II 3.18(3); *N* II 2.57(0)
4705.36	1	(2)	2	3	...	3	...	...	...	...	*O* II 5.36(8)
4707.98	...	...	...	1	...	1	...	...	...	...	.....
4709.54	...	...	...	...	...	1	...	...	...	...	*N* II 9.45(1)
4710.01	...	...	1	2	1	1	...	...	...	1	*O* II 0.04(5)
4713.24	5	4	4	7	3	10	7	×	×	4	*He* I 3.14(3); *He* I 3.37(1)
4716.46	...	...	...	...	...	2	...	...	1	1	*Si* III 6.66(5); *S* II 6.25(4)
4718.44	...	...	...	...	...	1	...	...	...	...	*N* II 8.43(2)
4726.59	...	...	...	...	...	...	...	...	2	...	.....
4803.31	...	...	...	...	...	1	...	...	...	...	*N* II 3.27(6)
4861.46	...	...	...	...	...	20	...	...	...	15	*Hβ* 1.33; [*O* II 1.03(3)]
4906.64	...	2	...	...	...	...	...	...	...	...	*O* II 6.88(5)
4921.96	...	(4)	...	...	...	...	...	...	...	4	*He* III 1.93(4)
4923.93	...	...	...	...	...	...	...	...	...	4	*Fe* II 3.92(10)

NOTES

3928.59 If 67 Ophiuchi is included, the wave-length is λ 3928.46.

3933.14 This line is blended with *Ca* II λ 3933 on most plates.

3933.60 Wave-length based upon β Orionis alone, since in other stars it is of interstellar origin.

3966.91 Measured twice on the same plate. Uncertain.

3968.46 Wave-length based upon β Orionis alone, since in other stars it is of interstellar origin.

3980.24 Probably blended with *S* II λ 3979.85(2).
3979.67 Ingram (*Physical Review*, **32**, 172, 1928) gives intensity 4.
3989.35 Uncertain.
3991.78 Uncertain.
4015.41 Attributed to *Ni* II by Baxandall.
4030.51 Several close *Al* II lines.
4045.00 There is a forbidden line of *He* I at λ 4045.17, but the identity is improbable.
4053.08 There is a forbidden line of *He* I at λ 4053.56, but the identity is improbable.
4064.89 Uncertain.
4067.04 Attributed to *Ni* II by Baxandall.
4069.81 10 Lacertae gives λ 4070.10 and may be a blend with some other lines.
4084.44 Perhaps a blend.
4097.26 *N* III probably predominates in 10 Lacertae, while *O* II predominates in β Canis Majoris.
4099.58 Uncertain. Very close to *Hδ*.
4124.56 Uncertain.
4129.02 Probably partly blended with *Si* II λ 4128.05.
4129.58 Uncertain.
4135.10 Uncertain.
4140.82 Rufus, Sawyer, and Paton (*Publications of the Astronomical Observatory of the University of Michigan*, **3**, 261, 1923) mention a line of *Si* at λ 4141.04. Ishida gives a forbidden line of *He* I at λ 4141.32.
4146.00 Uncertain.
4150.27 Uncertain. Perhaps *Al* III λ 4149.90 (3), etc.
4153.15 The wave-length is very sensitive to spectral type, because *O* II predominates in the early stars, while *S* II is more prominent in the later ones. The following wave-lengths are obtained separately for each subdivision:

O9	4153.18	B3	4153.03
B0	4153.09	B5	4153.07
B1	4153.33	B8	4153.08
B2	4153.23		

4159.45 Uncertain. Several other faint lines of *Al* II in this region.
4164.78 The wave-length does not agree with that of the *S* II line. The intensities suggest that the stellar line is mostly due to some other element.
4186.83 The wave-length in γ Pegasi is 6.44, agreeing roughly with *K* II 6.24(8), the strongest line of ionized potassium. The wave-length in 10 Lacertae is 6.96.
4188.74 Uncertain.
4199.84 The stellar line is probably a blend.
4202.89 Uncertain.
4207.15 Uncertain.
4220.07 Uncertain. Baxandall gives a line at λ 4220 in ζ Orionis.
4227.83 Several other faint lines of *Al* II in this region.
4232.04 Uncertain.
4243.84 Uncertain.
4263.81 Very doubtful. There is a line of *K* II at 3.40(7).
4275.68 There is a forbidden line of *He* I at 5.81, but the identity is improbable.
4276.95 Uncertain.
4277.55 The line of δ Ceti is probably blended with *S* II 8.51(4).

4278.60 The line of γ Pegasi is probably blended with *O* II 7.40(1).
4283.04 In β Canis Majoris and β Cephei *O* II predominates, the stellar wave-length being 3.29. In 67 Ophiuchi and ι Herculis *S* II predominates, and the stellar wave-length is 2.82; γ Pegasi gives an average wave-length of 3.01.
4286.24 This line seems to be blended in γ Pegasi with λ 4185.14.
4294.87 The wave-length in γ Pegasi is 4.47, the line being blended with *O* II 4.27.
4296.54 The line in γ Pegasi and τ Scorpii is probably not due to *Fe* II.
4299.88 The individual wave-lengths are very discordant, and there is some doubt as to the reality of this line.
4303.28 Possibly blended with *O* II 3.82(5).
4308.42 Uncertain. Baxandall gives in β Orionis 8.1.
4314.44 Very uncertain.
4325.64 The line in β Orionis, 5.84, is probably not due to *O* II.
4327.75 Very doubtful.
4332.68 Uncertain.
4347.47 Several other faint lines of *Al* II in this region.
4350.25 Somewhat uncertain.
4351.27 The line in 67 Ophiuchi, 1.50, is blended with *Fe* II 1.77(10).
4352.47 Blended with 3.84.
4358.06 Uncertain.
4370.31 Uncertain.
4395.88 In β Orionis, 5.43, and in 67 Ophiuchi, 5.76, this line is blended with *Ti* II 5.04(25).
4400.40 The identification is doubtful.
4404.26 Uncertain.
4405.13 Uncertain. Perhaps identical with *O* II 6.02(1).
4409.32 Uncertain.
4447.12 This line is blended with *O* II 8.21(6) on Yerkes single-prism spectrograms.
4466.29 In β Canis Majoris this line is blended with *O* II 5.40(4).
4491.03 This line is partly blended with *O* II 9.48(1).
4527.09 Uncertain.
4529.09 On some of the one-prism Yerkes plates this line is blended with *N* II 0.37(5).
4549.44 The wave-length in 10 Lacertae is 9.11; this line is not due to *Fe* II.
4553.93 Uncertain.
4572.12 The identification is uncertain.
4589.84 Very doubtful.
4597.80 Uncertain.
4602.00 This line is blended with *N* II 1.49(8).
4604.88 Uncertain.
4611.86 Uncertain.
4619.15 Rufus, Sawyer, and Paton (*op. cit.*, **3**, 261, 1923) give a line of *Si* at λ 4619.60.
4630.55 In τ Scorpii and β Cephei this line is blended with *Si* IV 1.38(3); and in β Orionis, with *Fe* II 9.33(4).
4634.50 Partly blended with unknown line at 3.75.
4638.71 Rufus, Sawyer, and Paton (*op. cit.*, **3**, 261, 1923) give a line of *Si* at λ 4638.36.
4649.23 This line is blended on some of the plates with *C* III 0.24(2).
4658.41 Uncertain.

4666.12 Very doubtful. Rufus, Sawyer, and Paton (*op. cit.*, **3**, 261, 1923) give a line of *Si* at λ 4665.76.
4678.28 Very doubtful.
4709.54 Uncertain. Rufus, Sawyer, and Paton give a line of *Si* at λ 4709.20.
4726.59 Somewhat uncertain.

No attempt has been made in Table II to provide accurate stellar wave-lengths which would replace those now being used in determinations of radial velocities. The purpose of this table is to serve as a basis for identifications. A comparison of my stellar wave-lengths with values obtained in the laboratory shows that the agreement is in general very satisfactory, the difference Star−Laboratory rarely exceeding a few hundredths of an Ångstrom unit. While the few lines showing greater discordances may be due in part to errors of measurement—either in the stellar spectra or in the laboratory plates—some of the discrepancies are doubtless caused by erroneous identifications or by the blending of adjacent lines on spectrograms of small dispersion.

The probable identifications are given in the last column of Table II. In the case of blends, the components are named in their probable order of importance. Laboratory lines in square brackets indicate improbable identifications. Additional information not contained in the body of the table may be found in the notes following it. If a line measured in a given star was subsequently regarded as suspicious, the estimate of its intensity was placed in parenthesis and its wave-length was not used in forming the mean. A line which was definitely seen, but not measured, is indicated by a cross (×) in the list of intensities.

The total number of lines in Table II is 379. Of these, approximately 150 are new. Nearly 130 lines have been newly identified with known elements. The number of unidentified lines is 73.

Table III contains a list of all elements found in stars of class B and the number of lines identified for each element. Group I gives the number of lines which are either free of blends or which appear as stronger components in blends with other lines; group II gives the number of lines which occur as fainter components in blends with other lines of the same ion; and group III gives the number of lines which are of uncertain identification or which are blended with stronger lines of other elements.

TABLE III

Element	Number of Lines			Source for Laboratory-Lines
	I	II	III	
H	6	.....	.....	A. Fowler, *Report on Series in Line-Spectra*, p. 90.
He I	21	6	3	*Ibid.*, p. 93.
He II	4	.....	.....	*Ibid.*, p. 95.
C II	8	7	7	A. Fowler and E. W. H. Selwyn, *Proc. Roy. Soc.*, A, **120**, 312, 1928.
C III	2	1	0	I. S. Bowen and R. A. Millikan, *Phys. Rev.*, **26**, 310, 1925.
N II	31	3	15	A. Fowler and L. J. Freeman, *Proc. Roy. Soc.*, A, **114**, 662, 1927
N III	10	.....	4	L. J. Freeman, *ibid.*, A, **121**, 318, 1928.
O II	101	25	21	A. Fowler, *ibid.*, A, **110**, 476, 1926.
O III	3	.....	1	*Ibid.*, A, **117**, 317, 1928.
Na I	.....	.....	.....	Outside of region measured.
Mg II	3	1	1	A. Fowler, *Series in Line-Spectra*, p. 120.
Al II	7	6	21	R. A. Sawyer and F. Paschen, *Annalen der Physik*, **84**, 1, 1927.
Al III	8	4	3	F. Paschen, *ibid.*, **71**, 142, 537, 1923.
Si II	5	.....	1	A. Fowler, *Phil. Trans. Roy. Soc. of London*, A, **225**, 1, 1925.
Si III	6	.....	.....	*Ibid.*
Si IV	5	.....	.....	*Ibid.*
P II	3	.....	9	I. S. Bowen, *Phys. Rev.*, **29**, 510, 1927.
P III	5	.....	.....	R. A. Millikan and I. S. Bowen, *ibid.*, **25**, 600, 1925; M. O. Saltmarsh, *Phil. Mag.*, **47**, 874, 1924.
S II	20	4	19	M. J. Gilles, *Recherches sur la structure du Soufre* (Paris, 1930), S. B. Ingram, *Phys. Rev.*, **32**, 173, 1928.
S III	10	.....	5	M. J. Gilles, *op. cit.*, S. B. Ingram, *op. cit.*
S IV	.....	.....	1	M. J. Gilles, *op. cit.*
K II	.....	.....	1	T. L. de Bruin, *Proc. Roy. Acad.* (Amsterdam), **29**, 713, 1926.
Ca II	2	.....	.....	A. Fowler: *Series in Line-Spectra*, p. 127.
Ca III	.....	.....	2	J. A. Anderson, *Ap.J.*, **59**, 76, 1924; I. S. Bowen, *Phys. Rev.*, **31**, 497, 1928.
Ti II	4	.....	2	H. N. Russell, *Ap.J.*, **61**, 223, 1925.
Cr II	1	.....	.....	T. Dunham, Jr., *The Spectrum of α Persei* (Princeton, New Jersey, 1929).
Fe II	20	.....	.....	H. N. Russell, *Ap.J.*, **61**, 223, 1925.
Ni II	2	.....	.....	A. G. Shenstone, *Phys. Rev.*, **30**, 255, 1927.

HYDROGEN

The hydrogen lines have been discussed at some length in a recent paper by Elvey and Struve.[1] The most interesting results are the relation between line-contour and Stark effect (illustrated in Figures 8 and 9 of that paper), and the pronounced tendency for stars of high absolute magnitude to have Balmer lines of comparatively small total intensity. The contours of $H\delta$ in 67 Ophiuchi (giant) and 85 ι

[1] *Astrophysical Journal*, **72**, 277, 1930.

Herculis (dwarf) are reproduced in Figure 1. 67 Ophiuchi is of a later spectral type than ι Herculis, and we should expect the hydrogen line to be stronger in the later type, but the opposite is observed. The same effect may be seen in the halftone illustrations (Plates XI–XV). The Balmer lines are especially sharp and narrow in β Orionis, while in γ Pegasi and in ι Herculis these lines have extended wings.

NEUTRAL HELIUM

Appearance of lines.—Neutral helium is represented in the B-type stars by the following lines:

He LINES IN B STARS

Triplets		*Singlets*	
DIFFUSE SERIES		DIFFUSE SERIES	
(5876)	$2p^3-3d^3$	4922	$2P-4D$
4472	$-4d^3$	4388	$-5D$
4026	$-5d^3$	4144	$-6D$
3820	$-6d^3$	4009	$-7D$
		3927	$-8D$
		3872	$-9D$
PRINCIPAL SERIES		PRINCIPAL SERIES	
(3888)	$2s^3-3p^3$	(5016)	$2S-3P$
		3965	$-4P$
SHARP SERIES		SHARP SERIES	
4713	$2p^3-4s^3$	(5048)	$2P-4S$
4121	$-5s^3$	4438	$-5S$
3867	$-6s^3$	4169	$-6S$
		4024	$-7S$
		3935	$-6S$
		3878	$-9S$

I have shown in several articles[1] that the members of the diffuse series of both singlet and triplet systems are broadened by the action of ionic fields in the reversing layers. The amount of broadening depends upon the density of the charged particles—ions and free electrons—and is therefore a measure of the star's luminosity. The evidence can now be extended to higher members of the series. In the reproduction of the spectrum of γ Pegasi, Plate XI, the lines

[1] *Ibid.*, **69**, 173, 1929; **70**, 85, 1929; **70**, 237, 1929.

λλ 3820, 3872, 3927, 4009, and 4026 appear conspicuously broad and diffuse, while λλ 3867, 3878, 3935, and 3965 are much narrower.

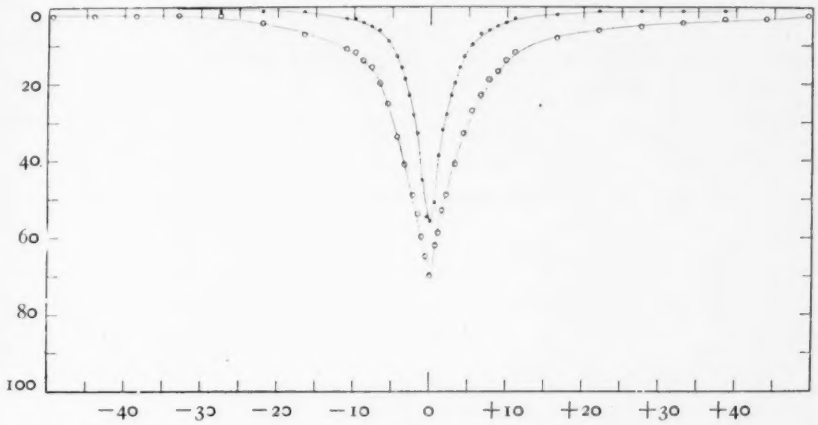


FIG. 1.—Contours of $H\delta$ in 67 Ophiuchi (dots) and 85 ι Herculis (circles). The ordinates are percentages of absorption and the abscissae are Ångstrom units.

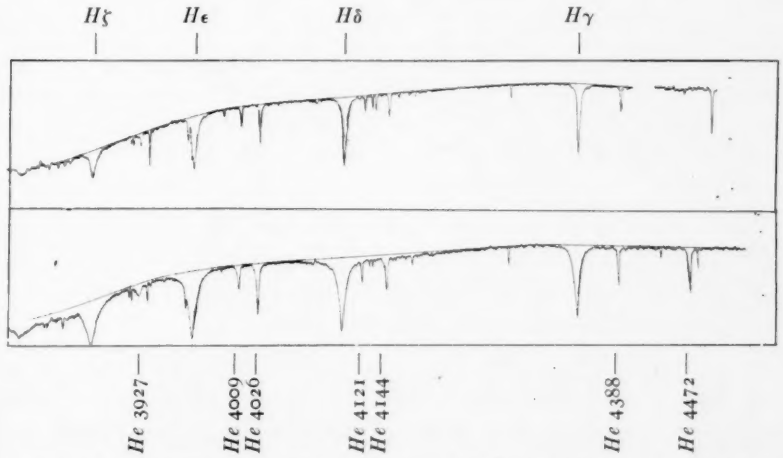


FIG. 2.—Microphotometric tracings of spectra of 67 Ophiuchi (top) and of 85 ι Herculis (bottom) made with the Coudé spectrograph of the Mount Wilson Observatory.

Microphotometric tracings of three plates are shown in Figures 2 and 3. The curve for γ Pegasi shows conspicuous widening in λ 3927 and λ 3872, while λ 3867 remains narrow and deep. In ι Herculis,

λ 3927 is very broad, and resembles the same line in γ Pegasi. On the other hand, 67 Ophiuchi, a giant, shows a narrow line at λ 3927, indicating little, if any, Stark effect.

The contours of the helium lines are shown in Figure 4 with two other lines for comparison. The relation of line-contour to spectral series is again well marked.

The broadening of the diffuse members can be ascertained also in λ 4388 and λ 4472, as I have already shown in earlier papers; and in agreement with J. Pauwen[1] the forbidden line of *He*, λ 4470, is seen

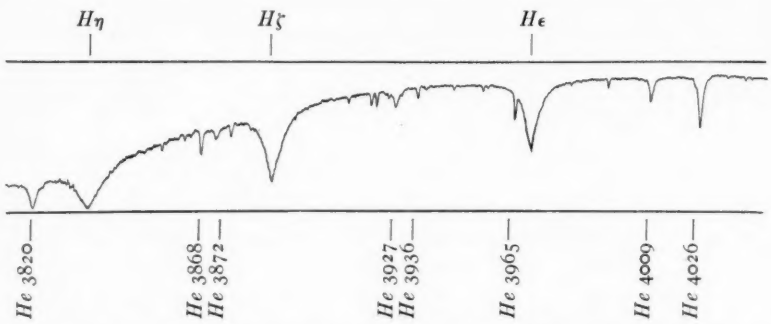


FIG. 3.—Microphotometric tracing of spectrum of γ Pegasi taken by Dunham with the Coudé spectrograph of the Mount Wilson Observatory.

to be broader than lines of other elements in the same region of the spectrum.

Wave-lengths of the He lines.—Table IV contains the wave-lengths of the helium lines as measured on the best available plates. The values agree, in general, with those in Table I, which are based upon more extensive material.

In a previous paper[2] I made the suggestion that the wave-lengths of the helium lines should be affected by the character of their Stark patterns. Certain lines are shifted by electrical fields toward the red, while others are not appreciably displaced, and a few are shifted toward the violet. It should be emphasized that the effect, if actually present, would be very small. As was shown in the case of the hydrogen lines,[3] the upper layers of the star's atmosphere super-

[1] *Ibid.*, **70**, 263, 1929. [2] *Ibid.*, **70**, 92, 1929.

[3] Elvey and Struve, *Astrophysical Journal*, **72**, 277, 1930.

impose upon the broadened wings a sharp and narrow core that is not affected by electrical fields. This core would, therefore, not be shifted at all; and it is only in the wings that a lack of symmetry might be detected.

The wave-lengths in Table IV indicate several cases of abnormally large differences Star − Laboratory. In the stars the line λ 3964 is

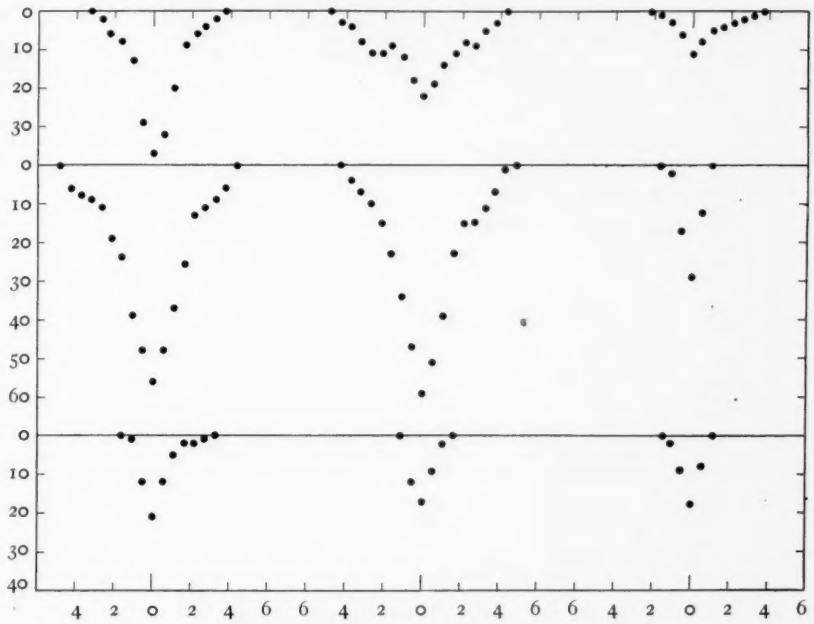


FIG. 4.—Contours of lines in γ Pegasi:

He 4009	*He* 3927	*He* 3872
He 4026	*He* 3820	*He* 3965
He 3872	*C* II 3919	*C* II 3921

Abscissae are Ångstrom units; ordinates are percentages of absorption.

systematically shifted toward the violet, while λ 4169 is shifted toward the red. This is in excellent agreement with my former results based upon single-prism Yerkes spectrograms.[1] However, I am not now certain that these shifts are actually due to Stark effect. The line λ 3964 falls upon the wing of the hydrogen line ε, as may be seen on the microphotometer tracings in Figures 2 and 3. It is to

[1] *Ibid.*, **70**, 94, 1929 (Table I).

be expected that a narrow line falling upon the wing of a broader one, would be shifted in wave-length away from the center of the broader line. The reason for this is the increased hydrogen absorption on the red side of the center of the helium line as compared to

TABLE IV

WAVE-LENGTHS OF *He* I

Laboratory	γ Peg; B246	γ Peg; R1599	γ Peg; C342	γ Peg; C345	ι Her; C325	ι Her; C325	67 Oph; C326	β Ori; R1768	β Ori; C122	β Ori; C129	β Ori; C161
3867.46 (2)											
3867.62 (1)			0.550	0.540							
3871.80 (1)			.936	.898							
3878.18 (1)				.57							
3926.53 (1)			.653	.686							
3935.91 (1)			.064	.210							
3964.727 (4)			.708	.718	0.565	0.647	0.704				
4007.71 forb.			.366			.501					
4009.27 (1)			.351	.347	.237	.369	.321		0.418		
4023.99 (1)			.106	.006		.883	.144				
4025.53 forb.			.165	.335		.353					
4026.189 (5)											
4026.358 (1)			.242	.241	.230	.216	.249		.225		
4120.812 (3)											
4120.981 (1)			.814	.857	.812	.870	.847		.787		
4143.77 (2)			.821	.822	.688	.834	.757		.663		
4168.97 (1)			.049	.038	0.104	.051	.968				
4387.928 (3)	0.056	0.951	.952	.935		.907	.914	0.997	.978	0.941	0.963
4437.549 (1)	.640	.580	.583	.581		.498	.601	.650		.731	
4469.97 forb.	.834	.980	.898	.964		.908					
4471.477 (6)											
4471.689 (1)	.559	.523	0.543	0.528		0.440	0.450	.619	.595	.544	.634
4713.143 (3)											
4713.366 (1)	0.168	0.196						0.393	0.231	.233	.275
4921.929 (4)										0.899	0.028

that of the violet side. I do not know whether this effect would explain the observational data quantitatively, but it may, at least in part, account for the shift. The line λ 4168.97 is a blend, and is therefore not suitable for a test of the shift resulting from Stark effect. The oxygen line λ 4169.23 has a laboratory-intensity of 4 and would tend to shift the blended line in the direction observed. While it is probable that some of the stellar wave-lengths of *He* I actually differ from those established in the laboratory, we shall leave the

question open as to whether or not appreciable shifts are caused by Stark effect.

Forbidden lines of He I.—The strongest forbidden line of *He* I, λ 4470 ($2p^3-4f^3$), has been measured in many stars, and a reliable stellar wave-length is therefore available for comparison with the laboratory results.

	Stellar Wave-Length
The whole material	4469.95
Best plates alone	4469.92

The best laboratory determinations, as quoted by Y. Ishida and T. Tamura,[1] are:

	Laboratory Wave-Length
Ishida and Tamura	4469.97
W. Tschulanowsky	4469.99
S. Foster	4469.92

The agreement is excellent, and there can be no doubt as to the correctness of my identification.

In addition, one other forbidden *He* I line has now been identified with certainty. Its stellar wave-length is 4025.32 ($2p^3-5f^3$), agreeing well with the following laboratory values:

$2p^3-5f^3$		$2p^3-5g^3$	
Ishida and Tamura	4025.58	Ishida and Tamura	4025.48
Tschulanowsky	4025.59	Foster	4025.53
Foster	4025.61		

The list of forbidden lines observed in the laboratory by Ishida and Tamura gives the approximate electric field at which each line first becomes visible. The lowest value, 100 volts, is assigned to the line λ 4470. Its strength in the stars is thus accounted for. There is another laboratory line, λ 4007.79 (2P−7P), which appears in a field of 140 volts. I have measured on several plates a line of mean wave-length λ 4007.53. The agreement is satisfactory, but the stellar line is so faint that its identity cannot be regarded as definitely established. It is much fainter than the other forbidden lines, λ 4470 and λ 4025, the latter of which is first seen in a field of <400 volts. At my request Dr. Ishida has checked his observations of

[1] *Scientific Papers of the Institute of Physical and Chemical Research*, **13**, 118, 1930.

λ 4007.79, and he finds that the limiting field strength, which was intended to give only a rough indication of the conditions, may be somewhat larger than 140 volts. However, he is convinced that it should not be higher than 400 volts/cm. Professor T. Takamine has also informed me that the line at λ 4007 is easily produced in the laboratory. More extensive data will be needed in order to decide whether the discrepancy between laboratory and stellar intensities is real.

Of the other forbidden lines listed by Ishida and Tamura, none would be expected to occur in stellar spectra. Nearly all of the stronger forbidden lines are so close to the permitted lines of *He* I that they are blended. A few doubtful coincidences in wave-lengths are recorded in Table II.

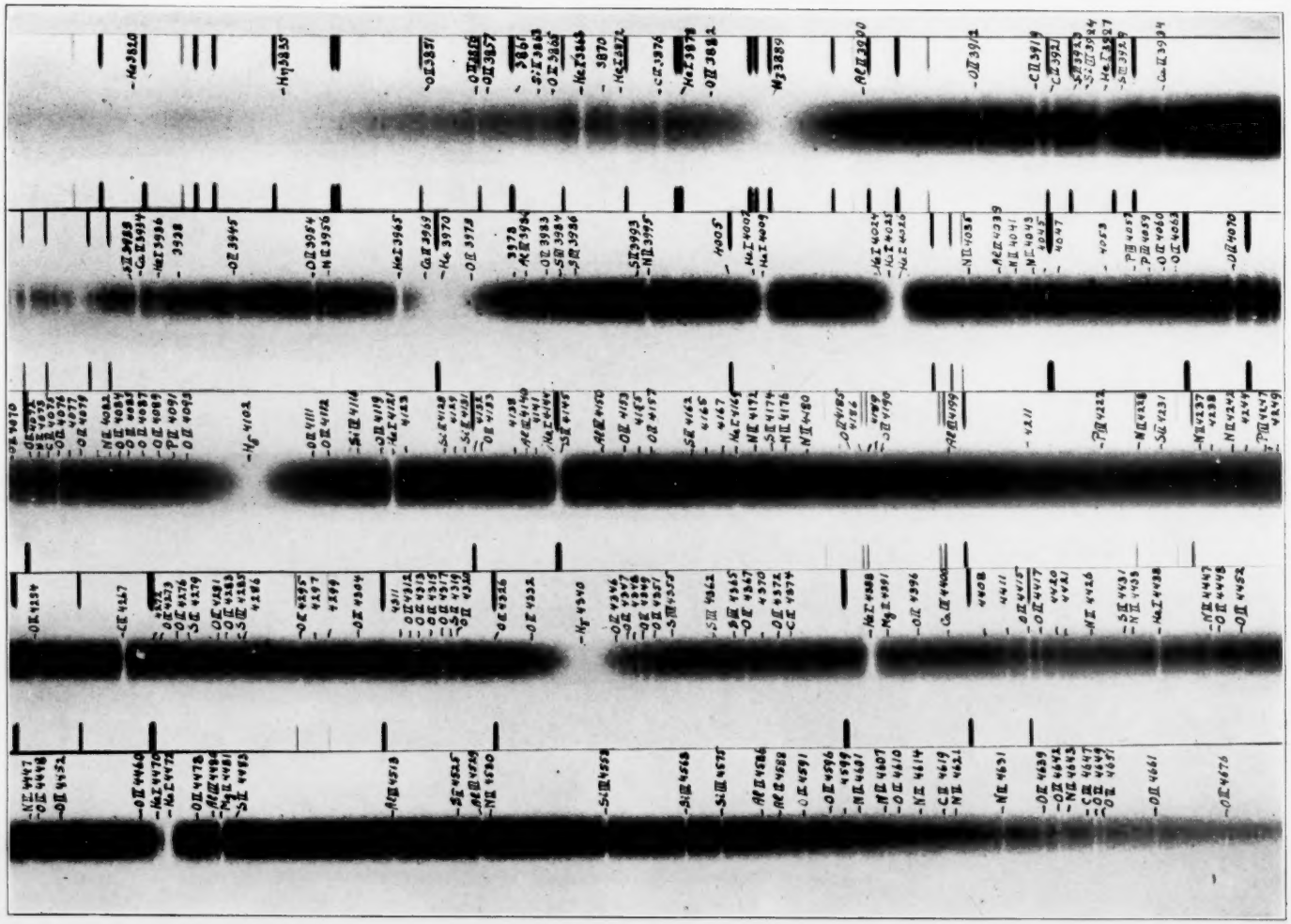
Intensities of He lines.—The relative intensities of the helium lines are not the same in all stars. This was first noticed some years ago[1] when I compared the estimated intensities of the triplet line λ 4472 with those of the singlet line λ 4388. I have now extended the material to include other lines, omitting all stars showing rapid rotation. Unfortunately, the number of stars with perfectly sharp lines is small, and it was therefore necessary to use a number of stars that had moderate rotational broadening.

The following results are intended to serve as a basis for a more precise spectro-photometric study of the helium intensities. I have therefore made estimates of the relative intensities of the following three pairs of lines:

$$\frac{4388\ (2P-5D)}{4472\ (2p^3-4d^3)}\ ;\qquad \frac{4009\ (2P-7D)}{4026\ (2p^3-5d^3)}\ ;\qquad \frac{4121\ (2p^3-5s^3)}{4144\ (2P-6D)}\ .$$

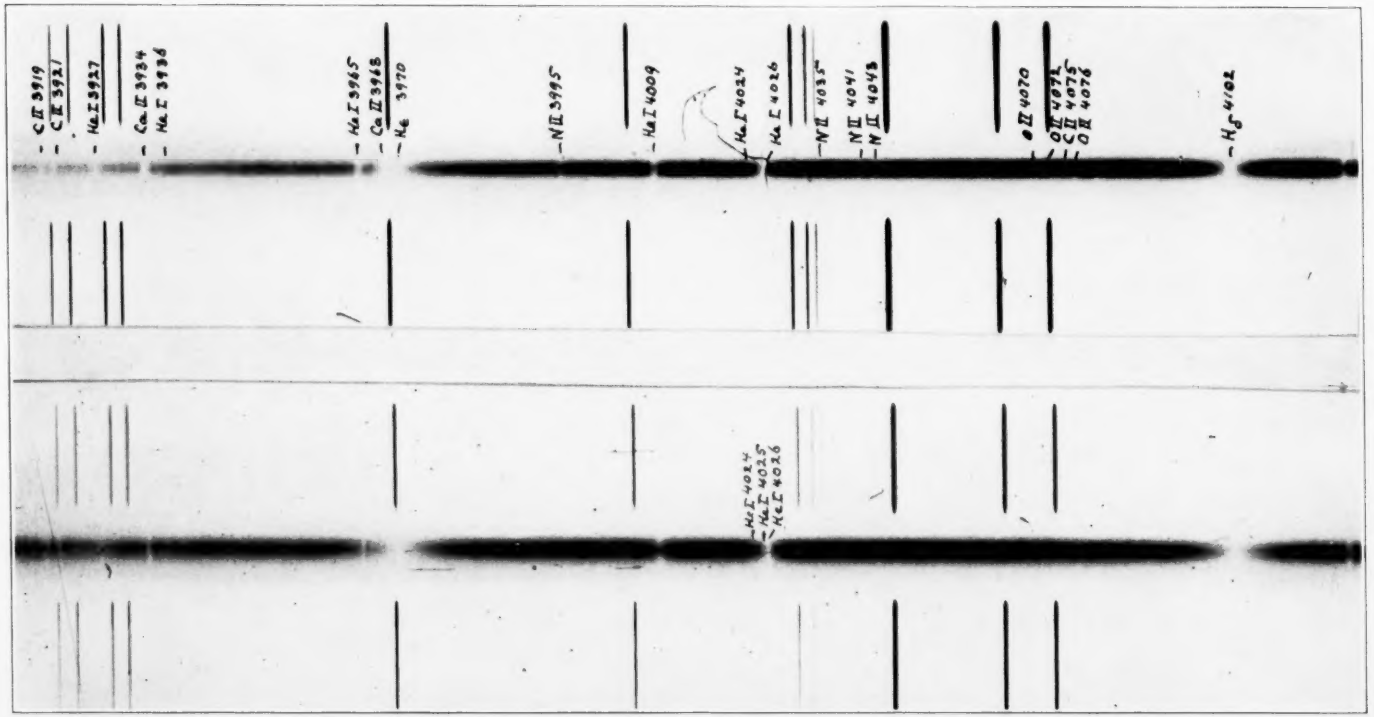
All stars have been reclassified according to the Harvard (and partly Mount Wilson) criteria. It will be seen that the agreement with the spectral types given in the *Henry Draper Catalogue* is (with a few exceptions) very good. I have attempted, furthermore, to make a rough classification within each spectral subdivision, according to absolute magnitude. This division into three groups—

[1] *Nature*, **122**, 994, 1928.



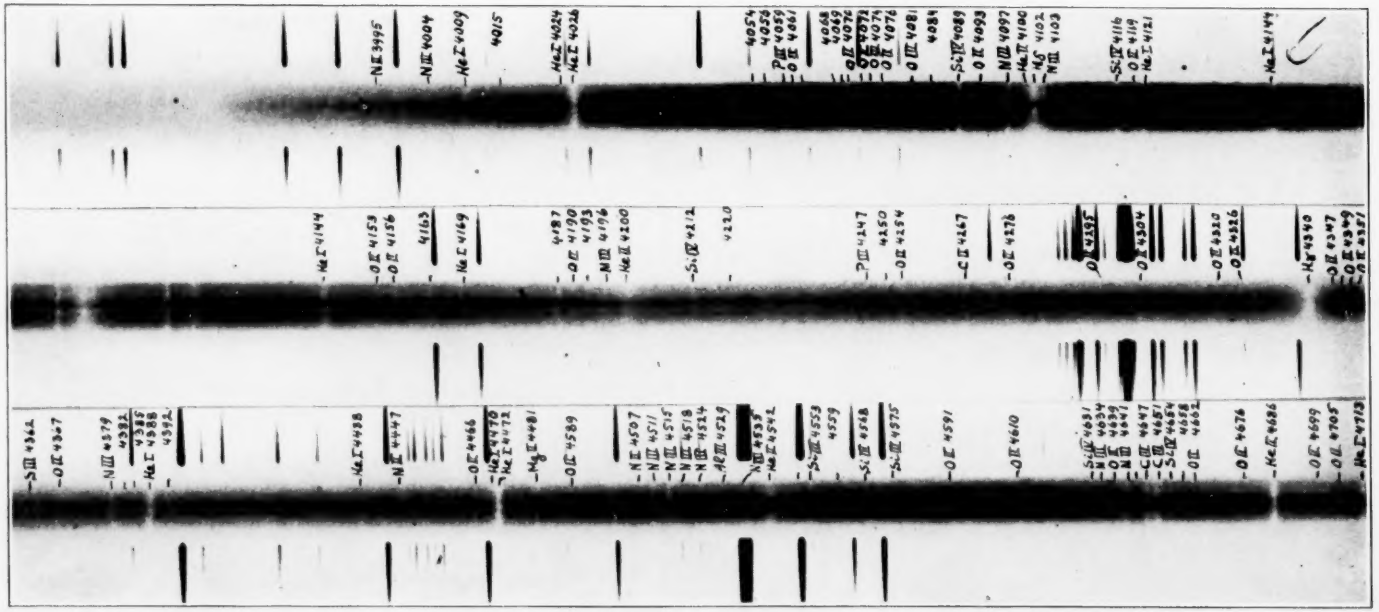
SPECTRUM OF γ PEGASI (B2) [PLATE C 345]

PLATE XII



Spectra of 67 Ophiuchi, B5p (top), and of ι Herculis, B3 (bottom) [Plates C 326 and C 325]

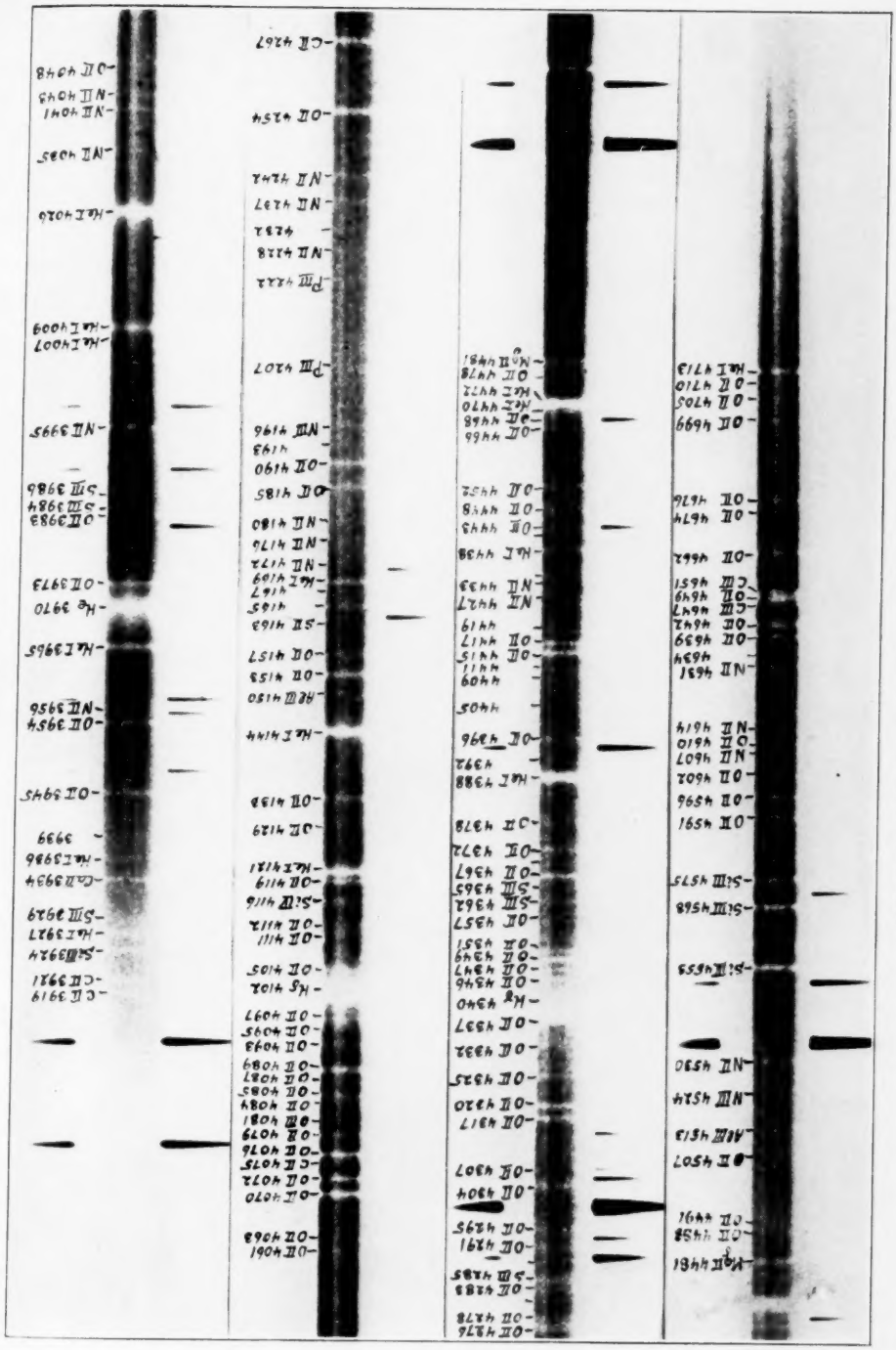
PLATE XIII



SPECTRUM OF 10 LACERTAE (O9) [PLATE IR 9506]

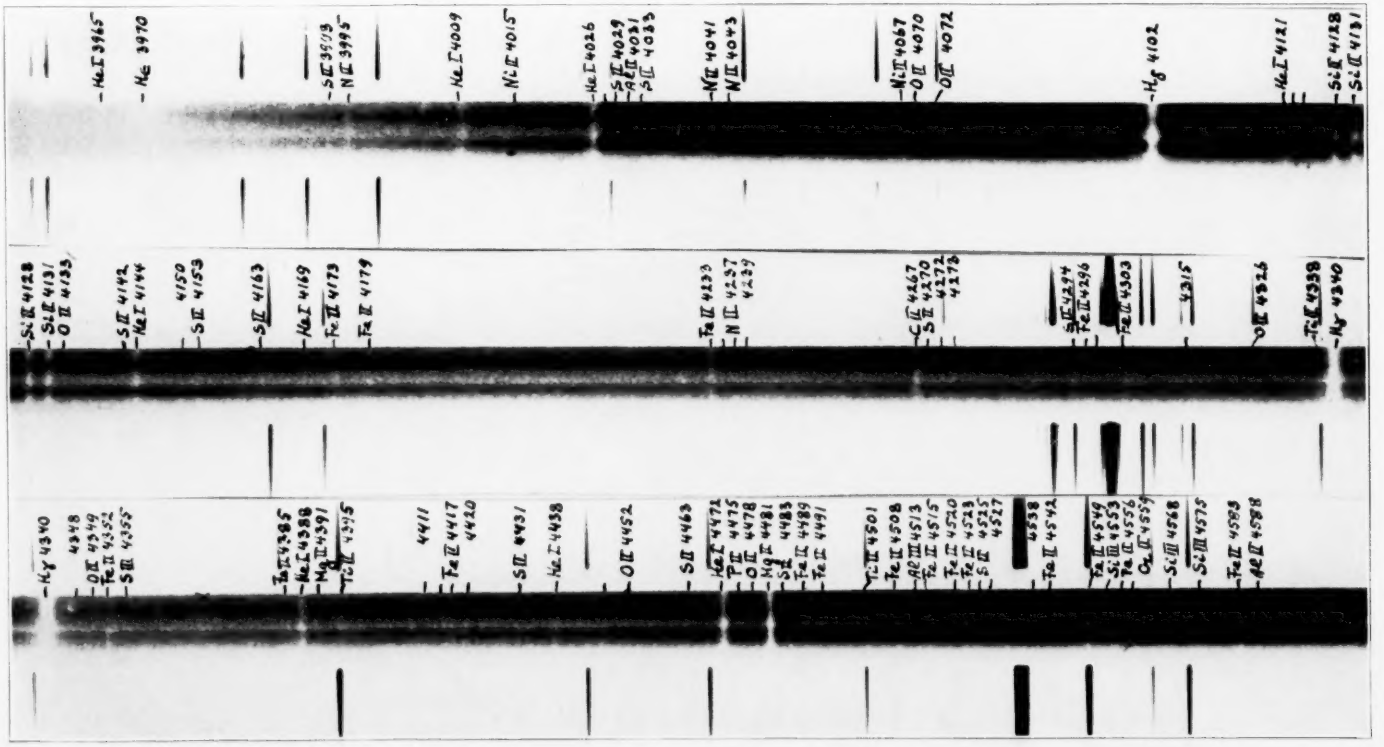
PLATE XIV

PLATE XIV



SPECTRUM OF β CANIS MAJORIS (B1) [PLATE IR 9307]

PLATE XV



SPECTRUM OF β ORIONIS (B8p) [PLATE IR 9515]

giants, intermediate stars, and dwarfs—is admittedly uncertain, and depends primarily upon the following criteria:

1. Presence or absence of wings in the Balmer lines (cf. γ Pegasi—dwarf; β Orionis—giant).
2. Presence or absence of forbidden line of *He* I at λ 4470.
3. Appearance of helium lines belonging to diffuse series.
4. Intensity of interstellar lines of *Ca* II.

Unfortunately, all four criteria are affected by rotational broadening, and this causes considerable uncertainty in the estimates. The latter are shown in Table V. The rotational broadening has been estimated on an arbitrary scale, zero indicating perfectly sharp lines and 10 standing for appreciable broadening. It should be remembered that stars with rapid rotation were not used in the work. The relative intensities of the lines are designated as follows:

=	equal
<	slightly fainter
<<	fainter
<<<	much fainter
<<<<	very much fainter

An inspection of the table gives the following results:

1. In spectral subdivisions B1, B2, and B3, all stars resemble rather closely the spectrum of γ Pegasi (Plate XI). The singlet lines are here nearly as strong as the triplet lines. It will be recalled that in the laboratory the singlets are usually much weaker than the triplets. There seems to be no difference, in these spectral classes, between giants and dwarfs.

2. As we proceed to the earlier spectral classes, Bo and O, the singlet lines fade out much more rapidly than the triplet lines. There are indications that this is more pronounced in the giants than in the dwarfs.

3. Among the later spectral classes, B7, B8, and B9, there is a similar fading-out of the singlets with respect to the triplets, but the effect is not as pronounced as in classes O and Bo. Supergiants of type B8, such as β Orionis, show less fading-out of the singlets than do stars of intermediate or low luminosity.

The halftone reproductions of stellar spectra in Plates XI–XV illus-

TABLE V

ESTIMATED RELATIVE INTENSITIES OF HELIUM LINES

Star	α	δ	Mg.	Harvard Sp.	Rot.	4388/4472	4009/4026	4121/4144
Spectrum O								
Giants:								
39 λ Ori	5^h30^m	$+9°52'$	3.7	Oe5	2	<<<	<<<<	>
19 Cep	22 2	+61 48	5.2	Oe5	4	<<	<<<<	=
Intermediate:								
15 SMon	6 35	+ 9 59	4.7	Oe5	4	<<	<<	
Dwarfs:								
30 τ CMa	7 15	−24 46	4.4	Oe5	6	<<	<<	>
10 Lac	22 35	+38 32	4.9	Oe5	0	<	<<	<
Spectrum Bo								
Giants:								
9 Cam	4 44	+66 10	4.4	Bo	4	<<	<<<	>
Intermediate:								
κ Cas	0 27	+62 23	4.2	Bo	3	<	<<<	>
+34° 980 Aur	5 10	+34 12	5.8	Bop	1	<	<<<	=
46 ε Ori	5 31	− 1 16	1.8	Bo	5	<	<<	>
48 σ Ori	5 34	− 2 39	3.8	Bo	7	<	<<	
53 κ Ori	5 43	− 9 42	2.2	Bo	4	<	<<	>
Dwarfs:								
34 δ Ori	5 27	− 0 22	2.5	Bo	8	<<<	<<<	>
36 υ Ori	5 27	− 7 23	4.6	B3	0	<	<<	=
23 τ Sco	16 30	−28 1	2.9	Bo	0	=	<<	=
59° 2334 Cep	21 9	+59 34	5.6	B2	3	<	<<	=
Spectrum B1								
Giants:								
44 ζ Per	3 48	+31 35	2.9	B1	3	=	<	<
47 ρ Leo	10 28	+ 9 49	3.8	Bop	3	=	<	<
Intermediate:								
37 φ^1 Ori	5 29	+ 9 25	4.5	Bo	0	=	<<<	=
Bond 708	5 30	− 5 29	6.3	Bo	4	=	<<	=
2 β CMa	6 18	−17 54	2.0	B1	1	=	<	<
15 CMa	6 49	−20 6	4.7	B1	3	=	<	<
21 ε CMa	6 55	−28 50	1.6	B1	1	=	<	<
20 σ Sco	16 15	−25 21	3.1	B1	5	=	<	<
1 Cas	23 2	+58 53	4.9	B1	0	<	<<	=
Dwarfs:								
6 λ Lep	5 15	−13 17	4.3	B1	1	<	<<	=
−1° 935 Ori	5 28	− 1 40	5.3	B2	0	=	<	<
−6° 1234	5 30	− 6 4	4.7	B1	0	=	<<	>
4 ξ^1 CMa	6 28	−23 21	4.4	B1	2	>	<	=
8 β Sco	16 0	−19 32	2.9	B1	7	<	<<	<
8 β Cep	21 27	+70 7	3.3	B1	0	=	<	<

TABLE V—*Continued*

Star	α	δ	Mg.	Harvard Sp.	Rot.	4388 4472	4009 4026	4121 4144
Spectrum B2								
Giants:								
62 χ^2 Ori	5^h58^m	+20° 8′	4.7	B2p	1	=	<	<
9 Cep	21 35	+61 38	4.9	B2p	2	=	<	<
Intermediate:								
40 o Per	3 36	+33 39	5.0	B2	4	=	<	=
α Pyx	8 40	−32 50	3.7	B2	0	=	<	<
42 θ Oph	17 16	−24 54	3.4	B3	1	=	<	<
12 Lac	22 37	+39 42	5.2	B2	0	=	<	<
Dwarfs:								
88 γ Peg	0 8	+14 38	2.9	B2	0	=	<	<
ζ Cas	0 31	+53 21	3.7	B3	0	=	<	<
82 δ Cet	2 34	− 0 6	4.0	B2	0	=	<	<
48 ν Eri	4 31	− 3 33	4.1	B2	0	=	<	<
3 π^4 Ori	4 46	+ 5 26	3.8	B3	2	=	<	<
22 o Ori	5 17	− 0 29	4.6	B3	0	=	<	<
24 γ Ori	5 20	+ 6 16	1.7	B2	3	=	<	<
102 Her	18 4	+20 48	4.3	B3	2	=	<	<
Spectrum B3								
Giants:								
55 Cyg	20 46	+45 46	4.9	B2	1	=	<	<
Intermediate:								
35 Ari	2 38	+27 17	4.6	B3	7	=	<	<
8 π^5 Ori	4 49	+ 2 17	3.9	B3	4	=	<	<
114 Tau	5 22	+21 51	4.8	B3	1	=	<	<
33 n^1 Ori	5 26	+ 3 13	5.5	B3	4	=	<<	<
−6° 1233	5 30	− 6 5	5.6	B1	1	=	<	<
126 Tau	5 36	+16 29	4.9	B3	8	<	<<	<
−1° 1004 Ori	5 36	− 1 11	5.0	B3	5	=	<	<
−19° 1407 CMa	6 14	−19 56	5.3	B3	3	=	<	<
20 η Lyr	19 10	+38 58	4.5	B3	1	=	<	<
8 Cyg	19 28	+34 14	4.8	B3	2	=	<	<
44 σ Aql	19 34	+ 5 10	5.2	B3	9	=	<	<
45 ω^2 Cyg	20 27	+48 37	4.9	B3	7	=	<	<
51 Cyg	20 39	+49 59	5.4	B3	2	=	<	<
81 π^2 Cyg	21 48	+48 51	4.3	B3	3	=	<	<
31 Peg	22 17	+11 42	4.9	B3p	5	=	<	<
6 Lac	22 26	+42 37	4.5	B3	4	=	<	<
Dwarfs:								
45 ϵ Cas	1 47	+63 11	3.4	B3	0	=	<	<
61 τ Ari	3 15	+20 47	5.2	B3	2	<	<<	=
29 Tau	3 40	+ 5 44	5.4	B3	6	<	<<	<
40 Tau	3 58	+ 5 10	5.3	B3	3	=	<	<
+50° 973 Per	4 13	+50 41	5.5	B3	4	=	<	<
65 ψ Eri	4 57	− 7 19	4.8	B8	2	<	<	<
11 Cam	4 57	+58 50	5.3	B3p	6	=	<	<
10 η Aur	5 0	+41 6	3.3	B3	7	<	<<	<
103 Tau	5 2	+24 8	5.5	B3	3	=	<	<

TABLE V—*Continued*

Star	α	δ	Mg.	Harvard Sp.	Rot.	4388 4472	4009 4026	4121 4144
Spectrum B3—*Continued*								
Dwarfs:—*Cont.*								
8 Lep	5^h19^m	−14° 1′	5.2	B3	1	=	<	<
67 ν Ori	6 2	+14 47	4.4	B2	0	=	<	<
−6° 1446 Mon	6 7	− 6 32	5.1	B3	0	=	<	<
1 ζ CMa	6 16	−30 1	3.1	B3	2	=	<<	<
10 Mon	6 23	− 4 42	5.0	B3	4	=	<	<
96 Her	17 58	+20 50	5.1	B3	0	=	<	<
11 δ^1 Lyr	18 50	+36 51	5.5	B3	5	=	<	<
+50° 2708 Dra	18 58	+50 23	5.2	B3	3	=	<	<
2 Cyg	19 20	+29 26	4.9	B3	8	=	<	<
80 π^1 Cyg	21 39	+50 44	4.9	B3	4	=	<<	<<
16 Lac	22 52	+41 4	5.5	B3	0	=	<	<
Spectrum B4, B5, B6								
Giants:								
25 χ Aur	5 26	+32 7	4.9	B1	0	<	<	=
20 ι CMa	6 52	−16 55	4.4	B5	1	=	<	<
24 o^2 CMa	6 59	−23 41	3.1	B5p	1	<	<<	<
67 Oph	17 56	+ 2 56	3.9	B5p	1	=	<	<
Intermediate:								
20 τ Ori	5 13	− 6 57	3.7	B5	0	<<	<<<	=
125 Tau	5 34	+25 50	5.0	B3	4	=	<	
−4° 1362 Mon	6 2	− 4 11	5.4	B3	0	<	<<	<
30 Sex	10 25	− 0 7	5.0	B5	5	<<	<<	
Dwarfs:								
29 π And	0 32	+33 10	4.4	B3	0	=	<<	>
42 π Ari	2 44	+17 3	5.3	B5	3	<	<	
33 τ^8 Eri	3 49	−24 54	4.8	B5	4	<<	<<<	
−16° 796 Eri	4 5	−16 39	5.4	B3	3	<<	<<	=
53d Per	4 14	+46 16	4.9	B3	0	<	<<	=
20 ρ Aur	5 15	+41 42	5.1	B3	5	<	<<<	=
133 Tau	5 42	+13 52	5.2	B5	4	<	<<	<
−8° 4010	15 29	− 8 51	5.2	B5	0	<<	<<	>
— 22 τ Her	16 17	+46 33	3.9	B5	0	<	<<	>
— 85 ι Her	17 37	+46 4	3.8	B3	0	=	<	<
3 Vul	19 19	+26 4	4.9	B5	3	<	<<	
2 ϵ Del	20 28	+10 58	4.0	B5	3	<	<<<	=
30 Peg	22 15	+ 5 17	5.4	B5	3	<	<<	<
2 Lac	22 17	+46 2	4.7	B5	4	<	<<	<
Spectrum B7, B8								
Giants:								
19 β Ori	5 10	− 8 19	0.3	B8p	0	<	<	=
13 μ Sgr	18 8	−21 5	4.0	B8p	2	<	<	=
+46° 3111 Cyg	20 52	+47 2	5.9	B8p	1	=	<<	>

TABLE V—*Continued*

Star	α	δ	Mg.	Harvard Sp.	Rot.	4388 4472	4009 4026	4121 4144
			Spectrum B7, B8—*Continued*					
Dwarfs.....								
+53° 102 Cas.	0^h31^m	+53° 37′	5.1	B5	4	<	<<	
46 ω Cas.....	1 48	+68 12	5.0	B8	3	<<	<<<	
89 π Cet.....	2 39	−14 17	4.4	B5	2	<	<<	
112 β Tau....	5 20	+28 31	1.8	B8	3	<	<<<	<
+16° 794 Tau.	5 26	+16 59	5.5	B9	2	<<	<<	
22 ζ Dra.....	17 8	+65 50	3.2	B5	0	<<	<<<<	<
18 Aql.....	19 2	+10 55	5.1	B8	5	<<		
21 Aql.....	19 9	+ 2 7	5.1	B8	0	<	<<	>
			Spectrum B9					
Giants:								
2 Hev Cam...	3 21	+59 36	4.4	B9p	0	<	<<	
4 Lac.....	22 20	+48 58	4.6	B8p	0	<	<<<	>

trate these points. Consider first the spectrum of γ Pegasi (Plate XI), a typical dwarf of class B2. Here λ 4388 is approximately equal in intensity to λ 4472, and λ 4009 is slightly fainter than λ 4026; similarly, λ 4121 is slightly fainter than λ 4144.

Consider next the two stars ι Herculis, a dwarf of class B4, and 67 Ophiuchi, a giant of class B5 (Plate XII). The relative intensities of λ 4009 and λ 4026 are the same in both stars, and are about equal to those of γ Pegasi. We conclude that there is no effect of absolute magnitude in the intermediate spectral types.

On the other hand, in 10 Lacertae, a dwarf of type O9 (Plate XIII), λ 4009 is extremely faint, while λ 4026 is fairly strong. However, since 10 Lacertae is characterized by much greater ionization than γ Pegasi, all its neutral helium lines are comparatively weak, which suggests that the relative intensity of singlets and triplets may be merely a function of the total intensity of the helium lines:

$$\frac{I_{singlet}}{I_{triplet}} = f(I_{triplet}) .$$

That this is not the whole truth may be ascertained from the spectrum of β Orionis (Plate XV), a supergiant of class B8. Here λ 4026

is weaker than it is in 10 Lacertae, but the intensity of λ 4009 is much greater. We conclude that while the ratio of the singlets to the triplets decreases on both sides of the maximum of the helium lines, this decrease is much more rapid on the side of high temperatures than on that of low temperatures.

It is difficult to explain this peculiar behavior of the helium lines. It is not due to Stark effect, since it does not depend upon absolute magnitude in the intermediate spectral types B1 to B5. For the same reason, pressure is excluded as a possible cause. It is probable that the peculiar intensities of the helium lines in stellar spectra are of a totally different nature than the changes in the relative intensities of singlets and triplets frequently observed in the laboratory[1] and invariably found to be related to the gas pressure.

The intensity of a stellar absorption line depends upon three factors: (1) the number of atoms in the upper and lower states which define the transition corresponding to the line, (2) the atomic absorption coefficient, and (3) the process by which the radiation is transmitted through the stellar atmosphere.

The number of atoms in any energy-level is given by the theory of ionization, provided we may consider the reversing layer as being in a state of thermodynamic equilibrium. This assumption has proved satisfactory in many applications of the theory, and we shall adopt it here, subject to further revision. We may then regard the number of atoms in the various states as defined uniquely by the temperature, the pressure, the potentials of ionization and of excitation, and by the statistical weights of the states. To be specific, consider the two lines λ 4026 ($2p^3 - 5d^3$) and λ 4009 ($2P - 7D$). The relative number of atoms in the two states $2p^3$ and $2P$ is:

$$\frac{N_1}{N_2} = \frac{\omega_1}{\omega_2} \cdot \frac{e^{\frac{-11{,}600\,E_1}{T}}}{e^{\frac{-11{,}600\,E_2}{T}}}$$

[1] L. S. Ornstein, H. C. Burger, and W. Kapuscinsky, *Zeitschrift für Physik*, **51**, 34, 1928; W. Hanle, *ibid.*, **56**, 94, 1929; W. Elenbaas, *ibid.*, **59**, 289, 1930; W. C. Michels, *Physical Review*, **36**, 1362, 1930.

where ω is the statistical weight of the state, E the excitation potential in volts, and T the temperature in degrees Centigrade. For the two lines under consideration,

$$E_1 = 20.81 \text{ volts},$$
$$E_2 = 21.12 \text{ volts}.$$

For a temperature of $T = 20{,}000°$ C, corresponding roughly to spectral type B2, we have

$$\frac{N_1}{N_2} = 1.2 \frac{\omega_1}{\omega_2}.$$

The numerical factor differs very little from unity. Even for a low temperature of only 10,000° C, it is equal to 1.4. We are therefore justified in concluding that the ratio N_1/N_2 is practically independent of the temperature and that we cannot account for the observed changes in the relative intensities of the two lines by attributing them to corresponding changes in N_1/N_2.

The process by which radiation is transmitted through the atmosphere of a star does not provide an explanation of the observations. Given the atomic absorption coefficient σ, the intensity in a line is simply

$$I = I_0 f(\sigma),$$

where the form of f is independent of outside conditions, such as temperature or pressure.

There remains the possibility that the atomic absorption coefficients corresponding to the two lines might vary with the conditions.[1]

[1] A. Unsöld has confirmed, in the case of hydrogen (*Zeitschrift für Astrophysik*, **2**, 199, 1931), the theory of V. Weisskopf and E. Wigner (*Zeitschrift für Physik*, **63**, 54, 1930; **65**, 18, 1930), according to which the absorption coefficient produced by radiation damping involves not only the oscillatory strength of the line but also the lives of the atom in its upper and lower states. In a recent paper Unsöld (*Zeitschrift für Astrophysik*, **3**, 94, 1931) has called attention to the fact that in helium the singlets have a very strong transition probability from state 2^1p to state 1^1s, while there is no corresponding probability for the triplets. Consequently, the functional relation of the absorption coefficient to wave-length is not the same for triplets and singlets, the latter being relatively the broader. This should, of course, result in a strengthening of the singlet lines, provided that all contours are widened predominantly by radiation damp-

If this should not prove to be the case, we should be tempted to doubt the assumption of thermodynamic equilibrium.

IONIZED HELIUM

Ionized helium is represented in Table II by the following lines: λ 4685.81 (Fowler series); and λλ 4541.63, 4199.87, 4100.08 (Pickering series). The lines λ 4859.36, near $H\beta$, and λ 4338.71, near $H\gamma$, were observed in several stars by H. H. Plaskett[1] but could not be seen on the Yerkes spectrograms because the latter were not sufficiently exposed to bring out faint lines in the immediate vicinity of the Balmer lines.

The two members of the Pickering series, λ 4542 and λ 4200, are appreciably broadened in many stars. The spectrum of 10 Lacertae, Plate XIII, illustrates this. On the other hand, the Fowler line λ 4686 is not appreciably broadened. We shall show that the broadening of the Fowler lines is caused by ionic Stark effect.

The *He* II ion is similar in structure to the atom of hydrogen, and its two series are expressed by the following relations:

$$\text{Fowler series:} \quad \nu = 4R\left(\frac{1}{3^2} - \frac{1}{[n']^2}\right);$$

$$\text{Pickering series:} \quad \nu = 4R\left(\frac{1}{4^2} - \frac{1}{[n']^2}\right).$$

The Fowler series originates from transitions between an upper energy-level of principal quantum number $n' = 4, 5, 6, \ldots$ and a lower level, $n = 3$; for the Pickering series the principal quantum numbers of the two levels are $n' = 5, 6, 7, \ldots$ and $n = 4$. In the presence of an electric field each of these states is split up into a number of separate levels characterized by the parabolic quantum

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ing. Unsöld suggests (private communication) that the observed differences in the ratios, singlets/triplets, may be due to the fact that radiation damping is not equally effective in all stars. Lines widened by thermal Doppler effect or by Stark effect would not be subject to the influence of the transition probabilities from the lower level. This question will be investigated further.

[1] *Publications of the Dominion Astrophysical Observatory* (Victoria), 1, 325, 1922.

numbers k_1 and k_2 and by the equatorial quantum number m. According to quantum mechanics,[1]

$$n=k_1+k_2+m+1 .$$

Hence,

$$0\leq k_1\leq n-1 ; \qquad 0\leq k_2\leq n-1 .$$

The displacements of the components in the Stark-effect pattern are given by the following formula,[2] in the usual notations:

$$\Delta\nu=\frac{3hF}{16\pi^2em_0}\left[n'(k_2'-k_1')-n(k_2-k_1)\right] .$$

For hydrogen the formula remains the same, except that the coefficient outside the brackets is twice as large.

The factor

$$\frac{3hF}{16\pi^2em_0}$$

is independent of the quantum numbers. Consequently, in comparing the relative displacements of corresponding components of two lines of *He* II, it is sufficient to consider the expression

$$X=n'(k_2'-k_1')-n(k_2-k_1) .$$

The maximum value of this expression for λ 4686 is obtained by substituting

$$n'=4; \qquad n=3; \qquad k_2'=3; \qquad k_1'=0; \qquad k_2=0; \qquad k_1=2 .$$

We then obtain

$$X_{\max}(\lambda\ 4686)=18 .$$

For λλ 4542 and 4200 the quantum numbers for maximum separation are:

	n'	n	k_2'	k_1'	k_2	k_1
4542	8	4	7	0	0	3
4200	10	4	9	0	0	3

[1] E. U. Condon and P. M. Morse, *Quantum Mechanics*, p. 82, 1929.

[2] R. Minkowski, *Handbuch der Physik*, **21**, 407, 1929.

Consequently

$$X_{\max}(\lambda\ 4542) = 68$$

$$X_{\max}(\lambda\ 4200) = 102\ .$$

If we assume, for simplicity, that the distribution of intensities among the various components of the Stark pattern is the same for all three lines, we obtain the following approximate ratios of the widths of the two Pickering lines to that of the Fowler line:

$$\frac{\text{Width}\ (\lambda\ 4542)}{\text{Width}\ (\lambda\ 4686)} = 3.8; \qquad \frac{\text{Width}\ (\lambda\ 4200)}{\text{Width}\ (\lambda\ 4686)} = 5.7\ .$$

This presupposes that the widths of all three lines are caused by Stark-broadening.

It is of interest to compare these values with those for the hydrogen lines:

	$H\alpha$	$H\beta$	$H\gamma$	$H\delta$	$H\epsilon$
n'	3	4	5	6	7
n	2	2	2	2	2
k_2'	2	3	4	5	6
k_1'	0	0	0	0	0
k_2	0	0	0	0	0
k_1	1	1	1	1	1
$\frac{1}{2}X_{\max}$	4	7	11	16	22

The values of $\frac{1}{2}X_{\max}$ should be directly comparable to those of *He* II. We see that the Stark-broadening of both Fowler lines exceeds that of $H\epsilon$, and would be equaled only by the higher members of the Balmer series, where blending and confluence of the wings make it impossible to study the contours in detail.

It may seem surprising that the total width of the hydrogen lines greatly exceeds that of the Pickering lines. This is due to the greater abundance of hydrogen in the reversing layer of the star. The maximum separation in the Stark effect, computed above, affects the atomic absorption coefficient and causes the latter to be much flatter and broader in the case of *He* II than in the case of *H*. But the contour of a line depends upon the optical depth, and consequently involves the number of atoms per cubic centimeter. There-

fore, the wide wings of the less abundant element *He* II may easily appear fainter than the wings of the more abundant *H*.

The appearance of the lines of *He* II in 10 Lacertae agrees qualitatively with the computed values. This star has probably a comparatively dense atmosphere, since the forbidden line of *He* I, λ 4470, is present and the wings of the hydrogen lines are broad and strong. It would be of interest to investigate the appearance of the Fowler lines in stars of low density. Unfortunately, there is appreciable rotational broadening in all the brighter stars of class O which, from the appearance of the hydrogen lines and from the strength of the interstellar line of *Ca* II, are suspected to be supergiants. In the star 9 Camelopardalis, however, where the rotational velocity is not large, the Fowler lines seem to be appreciably narrower than in 10 Lacertae.

An effect of broadening similar to that observed in 10 Lacertae was noticed by F. Paschen[1] in the laboratory for the higher members of the Fowler series, in the extreme ultra-violet part of the spectrum (λλ 2733, 2511, 2385, 2306, 2253, 2215, 2186, 2165). The broadening of the Pickering lines λ 4542 and λ 4200 has not, to my knowledge, been observed in the laboratory.

The relative intensities of the Fowler line λ 4686 to those of the Pickering lines seem not to be the same in all stars. While in 10 Lacertae λ 4686 is very strong, it is faint in 9 Camelopardalis.

SINGLY IONIZED NITROGEN

There are 49 lines of *N* II in Table II. Of these, 31 are free from blends or constitute the stronger components of blends. It is well known that many lines of *N* II are diffuse in the laboratory,[2] and W. E. Pretty[3] has shown that in some of the lines this broadening is accompanied by a shift in wave-length. The whole phenomenon may, according to Pretty, be due to Stark effect. F. E. Baxandall[4] believed that those lines which are hazy in the laboratory have also

[1] *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, p. 135, 1926.

[2] A. Fowler, *Proceedings of the Royal Society of London*, A, **114**, 664, 1927.

[3] *Proceedings of the Physical Society of London*, **41**, 442, 1929.

[4] *Researches on the Chemical Origin of Various Lines in Solar and Stellar Spectra* (Solar Physics Committee), p. 52, 1910.

a tendency to be hazy in stellar spectra, and I have suspected this also.[1]

Table VI shows the lines of intensity 3 or greater given by A. Fowler as being diffuse in the laboratory. A careful examination of these lines on my best plates fails to show any evidence of broadening (see γ Pegasi, Plate XI). Baxandall mentions also[2] the line λ 4530 as being broadened in stellar spectra. I believe that this may now be definitely ascribed to blending with *Al* III, 4529.09. The presence of

TABLE VI

HAZY LINES OF *N* II

λ	Int.	Remarks
4026.09	3n	Blended with *He* I 4026.19(5)
4035.09	4n	Blended with *O* II 4035.00(0)
4041.325	5n	Blended with *O* II 4041.20(0)
4043.54	3n	
4176.17	3n	
4227.83	3n	Blended with several lines of *Al* II
4236.98	6n	
4241.80	8n	
4432.71	6n	Blended with *S* II 4432.45(2)
4441.99	3n	
4677.93	3n	Not observed
4694.55	3n	

the latter line in stars having strong lines of *N* II (γ Pegasi and probably also γ Orionis) was not known to Baxandall.

Of the lines in Table II, there is only one, λ 4227.83, which, according to Pretty, shows an appreciable pressure-shift in wave-length. The stellar wave-length agrees exactly with the value found in the laboratory under low pressure. It is probable that the electric fields in stellar atmospheres are too small to produce measurable effects in nitrogen. While no numerical values are available, I believe that this is what should be expected from the amount of Stark-broadening observed in helium and hydrogen. The Stark effect decreases very rapidly with increasing atomic number.

SINGLY IONIZED OXYGEN

Singly ionized oxygen is represented in Table II by 147 lines, of which 101 are either entirely free from blends or constitute the major

[1] *Astrophysical Journal*, **70**, 96, 1929. [2] *Ibid.*

portions of blends, 25 appear as fainter components of blends with other lines of O II, and 21 are present as fainter components of blends with lines of other elements or are identified but tentatively.

The spectrum of O II has been analyzed by A. Fowler,[1] R. H. Fowler, and D. R. Hartree;[2] H. N. Russell;[3] I. S. Bowen;[4] C. Mihul;[5] and F. Croze and C. Mihul.[6] It consists of doublets and quartets and contains numerous intercombination lines. The intensities have not been determined accurately in the laboratory, but there are available estimates, on an arbitrary scale, by A. Fowler.[7] Many lines appear broad and diffuse in laboratory spectra (except when the pressure is very low). From analogy with *N* II, it seems probable that Stark effect is responsible for this broadening.[8]

The great number of O II lines in stellar spectra makes this element particularly suitable for detailed study, and for this purpose more accurate determinations of the laboratory intensities will be necessary. We shall at this time consider only two problems in connection with O II: the relative intensities of quartets, doublets, and intercombination lines in the laboratory and in stars; and the appearance in stellar spectra of lines which are diffuse in the laboratory.

We have seen before[9] that in the case of helium the singlets are, as a rule, much stronger in the stars than in the discharge tube. It is of interest to find whether such a difference exists in the case of O II. Table VII gives a summary of the estimated intensities of lines not seriously affected by blends. The difference between doublets and quartets is very slight. The number of lines due to intercombinations of the two systems is not sufficient to permit a reliable conclusion, but it is probable that there are no large differences between laboratory sources and stars.

Table VIII contains all lines of laboratory intensity 3 or greater between λ 3850 and λ 4700 which, according to A. Fowler, are diffuse (marked "*n*" in his paper). The only lines free from blends

[1] *Proceedings of the Royal Society of London*, A, **110**, 476, 1926.

[2] *Ibid.*, A, **111**, 83, 1926.

[3] *Physical Review*, **31**, 27, 1928.

[4] *Ibid.*, **29**, 231, 1927.

[5] *Comptes rendus*, **185**, 937, 1927.

[6] *Ibid.*, **185**, 702, 1927.

[7] *Loc. cit.*

[8] W. E. Pretty (*loc. cit.*) found reasons for attributing this type of broadening in *N* II to Stark effect.

[9] Page 249.

are λλ 4275, 4294, 4303, 4378. Our spectrograms (see especially γ Pegasi, Plate XI) give no definite evidence that any of these lines are broadened in the stars. There is a suspicion of faint wings on

TABLE VII

INTENSITIES OF O II LINES

LABORATORY INTENSITY	γ PEGASI						β CANIS MAJORIS					
	Doublets		Quartets		Intercombinations		Doublets		Quartets		Intercombinations	
	Int.	No.	Int.	No.	Int.	No.	Int.	No.	Int.	No.	Int.	No.
0	.....	.....	1.0	2	.....	.....	.....	.....	0.5	2	.....	.....
1	1.2	5	0.3	3	1.0	1	1.0	5	1.3	3	0.0	1
2	1.5	2	1.0	1	.....	.....	2.5	2	1.0	1	.....	.....
3	1.0	2	1.4	5	.....	.....	2.0	2	2.6	5	.....	.....
4	2.0	1	1.4	5	.....	.....	1.0	1	2.6	5	.....	.....
5	1.3	3	1.5	4	1.0	1	2.0	3	3.0	4	.....	.....
6	2.0	3	2.7	3	.....	.....	2.3	3	2.3	3	.....	.....
7	2.0	1	2.0	4	3.0	1	3.0	1	4.0	1	2.0	1
8	3.0	5	3.3	7	.....	.....	3.4	5	4.6	7	.....	.....
9	3.0	1	4.0	2	.....	.....	4.0	1	3.5	2	4.0	1
10	3.3	3	3.0	3	.....	.....	5.3	3	5.3	3	.....	.....

TABLE VIII

HAZY LINES OF *O* II

λ	Int.	Remarks
4060.58	3n	Blended with *O* II 4060.98(2)
4089.28	4n	Blended with *Si* IV 4088.86(10)
4097.25	4n	Blended with *N* III 4097.31(10); *O* II 4097.32(1)
4253.98	8n	Blended with *S* III 4253.51(6)
4275.52	4n	
4285.70	3n	Blended with *S* III 4285.00(5)
4294.82	3n	
4303.82	5n	
4342.00	4n	Blended with *Hγ*
4378.40	3n	
4491.25	3n	Blended with *Al* III 4490.90(2)
4609.42	4n	Blended with *N* II 4609.60(0); *O* II 4610.14(3)
4610.14	3n	Blended with *O* II 4609.42

either side of λ 4303, but these may be unreal or they may be due to other elements.

SILICON

The silicon lines are particularly suitable for a test of E. A. Milne's generalized ionization theory,[1] because the majority of B-type stars

[1] *Philosophical Transactions of the Royal Society of London*, A, **228**, 421, 1929.

show two stages of ionization and in some stars even three stages are represented.

Consider the two stars 67 Ophiuchi and ι Herculis. The first is a giant of Harvard class B5, while the second is a dwarf of class B3. Both stars show the lines of *Si* II, $\lambda\lambda$ 4128 and 4131, and of *Si* III, $\lambda\lambda$ 4553, 4568, 4575. In 67 Ophiuchi all five lines are appreciably stronger than in ι Herculis. This increase in the strength of lines belonging to two successive stages of ionization is unexpected and leads to interesting conclusions. According to the older theory, by Saha, it has usually been assumed that an increase in intensity of lines belonging to a certain stage of ionization can only be produced at the expense of some other stage. Now, in the case of 67 Ophiuchi and of ι Herculis, we are on the high-temperature side of the maximum of *Si* II, but on the low-temperature side of the maximum of *Si* III. A lowering in temperature alone would necessarily produce an increase in the intensity of *Si* II and a decrease in that of *Si* III. However, 67 Ophiuchi and ι Herculis differ not only in temperature but in absolute magnitude as well. Consider, for simplicity, stages 1, 2, and 3 of the silicon atom. We have, in the designations of Adams and Russell,[1]

$$\frac{N_3}{N_2}=3.28\times 10^{-7}\,\frac{T^{\frac{5}{2}}}{p_e}\,\frac{u_3}{u_2}\,e^{\frac{-11{,}600I_2}{T}}$$

$$\frac{N_2}{N_1}=3.28\times 10^{-7}\,\frac{T^{\frac{5}{2}}}{p_e}\,\frac{u_2}{u_1}\,e^{\frac{-11{,}600I_1}{T}}.$$

To these equations is added the basic assumption that $N_1+N_2+N_3$ remains constant throughout the sequence of stellar spectra. It is clear that an increase in N_3 must be compensated by a decrease in N_2 or in N_1, or in both. But in spectral classes B3 and B5 the lines of neutral silicon have disappeared, and it is difficult to escape the conclusion that an increase in N_3 must be compensated by a decrease in N_2. It would thus appear that the theory of ionization in its older form does not account for the simultaneous increase in the intensities of *Si* II and *Si* III.

We have implied in the preceding discussion that the depth of the reversing layer is the same in all stars. It remains to be seen whether a general increase in optical depth would account for the

[1] *Astrophysical Journal*, **68**, 11, 1928.

observations. A general increase in optical depth would enhance all lines, including those of hydrogen. As a matter of fact, the hydrogen lines decrease markedly in intensity as we pass from ι Herculis to 67 Ophiuchi. Similarly, the helium lines are appreciably stronger in ι Herculis. However, it is probable that this is due to Stark-broadening.

The difficulty may be overcome by applying the new theory of Milne. For the two stars in question we find surface gravities of approximately 10^3 and 10^4 cm/sec^2. Consequently, Milne's[1] computations and diagrams for silicon are immediately applicable. According to his scheme, the increase in intensity of *Si* II would be accounted for by the lower temperature of 67 Ophiuchi. Because of the so-called "null-effect" of Milne, there is no effect of absolute magnitude for *Si* II. On the other hand, the lines of *Si* III are very sensitive to absolute magnitude, and this not only would balance the expected decrease in intensity due to temperature but would even result in a strengthening of *Si* III in 67 Ophiuchi as compared to ι Herculis.

The same explanation may be applied to the two stars: γ Pegasi, a dwarf, and 9 Cephei, a giant. Both stars are classified as B2. In the giant, *Si* II and *Si* III are greatly strengthened, and I suspect that this is accompanied by a slight strengthening of the lines of *Si* IV. If the temperatures of these two stars were the same, *Si* II should not be strengthened. Since it actually is stronger in 9 Cephei than in γ Pegasi, we should conclude, on the basis of Milne's theory, that 9 Cephei has the lower temperature. It is difficult to test this in other lines without a detailed knowledge of their relation to absolute magnitude.

If we consider next two stars of type O9 to B0, namely 10 Lacertae and 9 Camelopardalis, the former a dwarf, the latter probably a giant, we obtain a similar result for *Si* III and *Si* IV. The lines of *Si* III are of about equal intensity in the two stars: Milne's null-effect on the high-temperature side of the maximum of *Si* III prevents a change of intensity depending upon absolute magnitude. For the same stars, however, *Si* IV is on the low-temperature side; consequently the giant 9 Camelopardalis has much stronger lines, λλ 4089 and 4116, than 10 Lacertae.

[1] *Loc. cit.*, p. 452.

The foregoing comparisons are purely qualitative and will in due course be replaced by actual measurements of line-contours. It is of interest, however, that Milne's generalized theory appears to give a much better representation of the observed intensities of the silicon lines than does the older form of the ionization equations, which dealt exclusively with concentrations without reference to atmospheric depth.[1]

An interesting side-issue is the determination of a lower limit for possible differences in total abundance of silicon in the atmospheres of stars of any one spectral subdivision. It is significant that whenever stars of similar spectral type show different intensities of the silicon lines, Milne's theory provides a satisfactory explanation. If differences in abundance were large, we should expect that stars, assigned to the same spectral class on the basis of their oxygen or helium lines, might have totally different lines of silicon. In classifying the spectra for Table V, I have attempted to use as many criteria as possible. The question arises whether these spectra are homogeneous except for effects of absolute magnitude. Let us consider, for example, the dwarfs of spectral class B2, listed in Table V. The lines *Si* II λ 4128 and λ 4131 are not sensitive to small differences in absolute magnitude and are therefore suitable for this test. An intercomparison of these lines in all eight stars shows very small differences, which may perhaps be explained by small errors in the classification. However, to obtain a lower limit, let us suppose that all the changes noted are caused by differences in the abundance of silicon. In order to obtain numerical values, we note that the differences in the intensities in no case exceed the normal difference between the two multiplet lines λ 4553 and λ 4574 in any one star, and that in most cases they are well below the difference between λ 4553 and λ 4568. The lines λλ 4553, 4567, and 4574 form a triplet of *Si* III (2^3s-3^3p), and have the following relative intensities in emission: 5:3:1. These values are proportional to the numbers of atoms contributing to the three component lines.[2] It is reasonable to conclude that individual stars do not differ in abundance

[1] See also *Publications of the American Astronomical Society* (Forty-sixth meeting), **7**, No. 2, 1931.

[2] Struve and Elvey, *Astrophysical Journal*, **72**, 267, 1930.

of silicon by more than a ratio of 5:1 and that the ratio is usually well below 2:1.

OTHER ELEMENTS

Of the other elements identified in B-type stars *S* II leads with 43 lines. *S* III has 15 lines, and in 10 Lacertae there is a line which is probably a blend of a strong line of *S* IV, not heretofore identified, with a line of *N* III. Nearly all of the sulphur lines measured by F. E. Roach[1] have been here confirmed. The new identifications of sulphur were made with the help of a complete laboratory study of this element recently published by M. J. Gilles.[2] Many new lines could be attributed to *C* II, the total being 22. *Al* III, which I found recently in several B-type stars,[3] was confirmed with a total of 15 lines. Many lines not previously identified could now be attributed to *Al* II. The presence of *P* III was confirmed[4] and several other lines were attributed to *P* II. *O* III is represented by 4, and *N* III by 14 lines. *Ca* II is represented by the H and K lines. There is a possibility that two lines which seem to reach maximum intensity in β Cephei are due to *Ca* III. *Mg* II is strong throughout the sequence of B stars; *Ti* II and *Fe* II appear only in the later subdivisions. Table II contains one line, λ 4559, which Baxandall has attributed to *Cr* II, and two lines which he has attributed to *Ni* II.

An attempt has been made to detect other elements which, from their excitation and ionization potentials, might be expected to be present in B-type stars. There is one line, λ 4186.83, which is perhaps identical with λ 4186.24, the strongest line of ionized potassium within the range of spectrum observed by me; but the wave-lengths do not agree as closely as might be desired. The absence of other lines of *K* II (ionization potential 31.6 volts)[5] is surprising in view of the abundance of this element in the crust of the earth. No evidence was found for *B* II (ionization potential 24.2 volts) or for *Cl* II (ionization potential 22.5 volts). It is also surprising that stellar

[1] *Ibid.*, **72**, 191, 1930.

[2] *Recherches sur la structure des spectres atomiques du soufre.* Paris, 1930.

[3] *Astrophysical Journal*, **71**, 67, 1930.

[4] *Ibid.*, **71**, 150, 1930.

[5] T. L. de Bruin, *Proceedings of the Royal Academy* (Amsterdam), **29**, 713, 1926.

spectra contain no lines of *Ti* III (ionization potential 27.6 volts),[1] *Ti* IV (ionization potential 43.1 volts),[2] or *Na* II (ionization potential 47 volts).[3]

The number of unidentified lines in Table II is not large, but there are some which are fairly strong and which appear in many stars. Their behavior with respect to type, and consequently with respect to temperature, may be ascertained from the intensities in Table II. The most conspicuous unidentified line is λ 4419.62. It is observed throughout nearly the whole sequence of B stars and reaches maximum intensity at or near B2.

I am greatly indebted to Dr. W. S. Adams for permission to use the Coudé spectrograph attached to the 100-inch telescope of the Mount Wilson Observatory and to Dr. Th. Dunham, Jr., for two plates of γ Pegasi which have an important part in this investigation. Various members of the staff of the Yerkes Observatory have co-operated in the observations with the 40-inch refractor, notably Messrs. W. W. Morgan, H. F. Schwede, F. E. Roach, and F. R. Sullivan. Acknowledgment is also due, for various suggestions, to Professors T. Takamine and Y. Ishida, of Tokyo, and to Dr. R. Ruedy, of Toronto.

YERKES OBSERVATORY
May 15, 1931

[1] H. N. Russell and R. J. Lang, *Astrophysical Journal*, **66**, 13, 1927.

[2] *Ibid.*

[3] F. H. Newman, *Philosophical Magazine*, **5**, 150, 1928.

A NUMERICAL METHOD OF DETERMINING THE SPACE DENSITY OF STARS[1]

By FREDERICK H. SEARES

ABSTRACT

The article develops and illustrates a numerical method of solving by approximations the fundamental integral equation of stellar statistics for the determination of the *space-density function*. The *unknowns are* the *relative densities d* in successive spherical shells whose thickness corresponds to a constant increment in the logarithm of the distance. The first approximation is obtained by using the fact that the *star ratio* for magnitudes $m+1$ and m, which is known from star counts, *bears a simple relation* to the *weighted mean d* for the shells which contribute stars of magnitude m. The *final approximation* is made by calculating the effect of *changing each d* in succession by *a standard amount*, and then combining the series of differential changes, each multiplied by an appropriate factor, in such a way as to reproduce the residuals left by the preceding approximation.

No assumption is made as to the form of the *density function* or the frequency functions for apparent and absolute magnitudes. The *luminosity function may vary* with the distance. The illustrative *example*, solved with a luminosity function which is the same at all distances, *leads* to a rapid *change in density* close to the sun that is *not supported* by other evidence. The *discordance* is *discussed* in *Contribution* No. 437.

I. INTRODUCTION

The determination of the number of stars per unit volume at different distances from the sun depends on the integral equation

$$N(m)dm=\omega dm\int_0^\infty \phi(M)D(\rho)\rho^2 d\rho\ , \qquad (1)$$

in which ω is a constant and N and ϕ are distribution functions for apparent (m) and absolute (M) magnitudes, respectively. D, here regarded as depending only on the distance ρ, is the required density function. In case space absorption is not negligible, D represents a fictitious distribution of stellar density, from which the true distribution Δ may be found, provided that the absorption function is known.[2]

Most of the solutions proposed for equation (1) depend on some simplifying assumption as to the form of the frequency functions for

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 436.

[2] *Mt. Wilson Contr.*, No. 428; *Astrophysical Journal*, **74**, 91, 1931.

N and ϕ or of the density function itself, and none gives a complete representation of the observational data. Schwarzschild's[1] general solution of integral equations of the form of (1) is mathematically sufficient, but requires developments in Fourier's coefficients for N and ϕ and leads to extensive calculations if these functions are to be accurately represented. Much simpler is his special solution—a quadratic exponential in log ρ for D—which corresponds to the case of normal frequency functions for N and ϕ. This solution has been much used because it permits a simple analytical treatment of many problems of stellar distribution. An obvious defect, which implies that N and ϕ cannot be exactly represented by Gaussian functions, is that close to the sun the calculated densities are much too small, and zero at the sun itself.

In itself, this failure is not very serious, because the density near the sun may be found in another way. A greater objection is the inflexibility of the method, which requires a Gaussian distribution for the star counts. This condition is satisfied, and then but approximately, only when the irregularities in N arising from the galactic star clouds and from other stellar aggregations have been smoothed out by averaging over many square degrees; but this procedure of course also eliminates all irregularity from the calculated density function, and the true distribution of density thus escapes us.

A method which avoids these difficulties begins by resolving equation (1), for each value of m, into elements corresponding to moderate intervals in ρ (actually, constant intervals in log ρ are used). Since the frequency ϕ and the volume factor $\omega\rho^2 d\rho$ are known for each element, we thus have, instead of (1), a group of linear simultaneous equations in which the mean densities, D_1, D_2, D_3, ... , in a series of successive shells, are the unknowns to be determined. The practical difficulties of dealing with fifteen or twenty equations involving an equal number of unknowns are overcome by a method of approximation which leads to a satisfactory representation of the star counts.

It must be noted, however, that the density problem involves an inherent uncertainty which apparently cannot be overcome by any statistical method of treatment. Because of the enormous range in

[1] *Astronomische Nachrichten*, **185**, 81, 1910.

the intrinsic brightness of stars, the apparent magnitudes in a cluster or star cloud show a range of 15 or 20 magnitudes. The sharp narrow maximum in the density function caused by the presence of the cloud therefore appears in the star counts as a very broad flat maximum. The location of such a cloud is therefore always difficult. For a remote cloud the maximum itself may not be accessible to observation; the solution then becomes highly sensitive to systematic error in the magnitude scale. For a succession of

TABLE I

MEDIAN M CORRESPONDING TO MEDIAN VALUES OF m AND OF LOG ρ

MEDIAN LOG ρ	MEDIAN m					
	0.5	1.5	2.5	3.5	4.5	5.5
−0.1	+1	+2	+3	+4	+5	+6
+0.1	0	+1	+2	+3	+4	+5
0.3	−1	0	+1	+2	+3	+4
0.5	−2	−1	0	+1	+2	+3
0.7	−3	−2	−1	0	+1	+2
0.9	−4	−3	−2	−1	0	+1
1.1	−5	−4	−3	−2	−1	0
1.3	−6	−5	−4	−3	−2	−1
+1.5	−7	−6	−5	−4	−3	−2

clouds seen in the same direction the solution is practically indeterminate. For example, the superimposed maxima produced by clouds at distances of 1600, 2500, 4000, and 6300 parsecs would be separated by intervals of only one magnitude, and without supplementary information the resultant curve could not be resolved into its components. The point at which the nearest of a group of clouds begins to affect the density function can be located, but what lies beyond is likely to remain doubtful.[1] Although the outlook for a complete solution of the density problem is not hopeful, the sensitiveness of the calculated densities to small changes in the values of N makes it the more important that the method used be one which gives the closest possible representation of the star counts.

II. THE METHOD

It is advantageous to introduce as unknowns, not the values of D itself as suggested above, but the logarithms of the ratios of D for

[1] Cf. *Mt. Wilson Contr.*, No. 347, Introduction; *Astrophysical Journal*, **67**, 125, 1928.

successive shells at regularly increasing distances from the sun. For these unknowns a first approximation is easily found, and the range in their numerical values always remains moderate. For dm in equation (1) we use an interval[1] of 1 mag., and for $d\rho$ the corresponding interval of 0.2 in log ρ. The median values of M found from

$$M = m - 5 \log \rho \tag{2}$$

(Kapteyn's zero-point for M; ρ in parsecs) for successive median values of m and log ρ are shown in Table I, which is to be thought of as extending downward and to the right as far as necessary. The lines of this table correspond to shells having boundaries at log $\rho =$ 0.0, 0.2, 0.4, ... The volumes of the shells included within the angular area ω are

$$\omega \rho^2 d\rho = V_{-1},\ V_1,\ V_3 \cdots, \tag{3}$$

where the subscripts are 10 times the median log ρ of the respective shells, and where, further,

$$V_1 = \sigma V_{-1},\ V_3 = \sigma V_1,\ V_5 = \sigma V_3 \cdots, \qquad (\log \sigma = 0.6)\ . \tag{4}$$

If the mean densities of the shells be denoted by D_{-1}, D_1, D_3, ... , equation (1) may be replaced by

$$\left.\begin{aligned} N(m) &= \phi_a D_{-1} V_{-1} + \phi_b D_1 V_1 + \phi_c D_3 V_3 + \cdots \\ \text{or} \qquad N(m) &= n_{-1} + n_1 + n_3 + \cdots \end{aligned}\right\}, \tag{5}$$

where

$$\phi_a = \phi(m + 0.5)\ , \qquad \phi_b = \phi(m - 0.5)\ , \qquad \phi_c = \phi(m - 1.5)\ , \ldots\ .$$

Equations (5) represent the total number of stars in the interval $m \pm 0.5$, the individual terms on the right denoting the contributions of the successive shells to this total. For magnitude $m+1$ we have, similarly,

$$N(m+1) = \cdots + \phi_a D_1 V_1 + \phi_b D_3 V_3 + \phi_c D_5 V_5 + \cdots\ . \tag{6}$$

[1] In special cases it may be desirable to use intervals of 0.5 in m and M, and of 0.1 in log ρ. The adaptation of the formulae to this case will be obvious.

Introduce now the density ratios, d, defined by

$$D_1 = d_0 D_{-1}, \qquad D_3 = d_2 D_1, \qquad D_5 = d_4 D_3 , \ldots . , \tag{7}$$

where d_0, d_2, ... refer to the distances log $\rho = 0.0$, 0.2, ... With the aid of (4) and (7), equation (6) then becomes

$$N(m+1) = \sigma(\cdots n_{-1} d_0 + n_1 d_2 + n_3 d_4 + \cdots) , \tag{8}$$

and we have the important relation

$$\frac{N(m+1)}{\sigma N(m)} = \frac{n_{-1} d_0 + n_1 d_2 + n_3 d_4 + \cdots}{n_{-1} + n_1 + n_3 + \cdots} = \delta_m . \tag{9}$$

This equation states that *the star ratio for magnitudes* $m+1$ *and* m, *divided by the volume ratio* σ, *is equal to the weighted mean* (δ_m) *of the relative densities* (d_s) *for the shells which contribute stars of magnitude* m. The star counts therefore give the values of log δ_m, log δ_{m+1}, ... The problem is to find from the weighted means the values of log d_0, log d_2, ... Results for log D_{-1}, log D_1, ... may then be found by a simple summation of the values of log d.

For the ideal case in which all the distribution functions are quadratic exponentials (Schwarzschild's simplified solution), it can be shown that log δ_m equals the value of log d at the distance corresponding to the mean log ρ for the stars of magnitude m. Although this simple relation does not hold in practice, it suffices to give a first approximation for the values of log d_s. This we obtain by plotting the mean log ρ for stars of magnitude m, $m+1$, ... against the values of log δ_m, log δ_{m+1}, ... From the curve thus defined, with the arguments log $\rho = 0.0$, 0.2, ... , we read the required approximations for log d_0, log d_2, ...

The mean log ρ for stars of magnitude m depends, however, on the density function, and we therefore need at the outset an approximation for D. To illustrate the procedure it is desirable to introduce at this point an example.

III. AN EXAMPLE

For numerical illustration we use the star counts for latitude 0° given in Table V of *Groningen Publication* No. 27 and afterward

slightly modified by van Rhijn.[1] Smoothed values of log $N(m)$, which refer to the limits $m \pm 0.5$ and an area of 1 square degree, are in the second column of Table II. The results for log δ_m found by equation (9) are in the fourth column of the table. Thus,

$$\log \delta_{0.5} = 6.953 - 6.411 - 0.6 = -0.058 .$$

In addition to these fundamental data, we require values of the luminosity function $\phi(M)$. For these we use the results of Kapteyn and van Rhijn,[2] for which the interval is $M \pm 0.5$.

TABLE II

STAR COUNTS AND VALUES OF LOG δ_m

MEDIAN m	OBSERVED LOG $N(m)$ 1 SQ. DEG.	O−C LOG $N(m)$	OBSERVED LOG δ_m	O−C FOR LOG δ_m (UNIT = 0.001)					
				(2)	(3)	(4)	(5a)	(5)	Final
0.5	6.411	+2	−0.058	+ 3	− 3	− 3	−4	−3	−3
1.5	6.953	−2	.061	7	− 1	− 2	−1	−1	−1
2.5	7.492	−3	.065	9	− 1	− 2	0	0	+1
3.5	8.027	−2	.069	16	+ 3	0	+2	+3	+3
4.5	8.558	0	.074	20	+ 4	+ 1	+2	−1	−1
5.5	9.084	−1	.082	30	+ 8	− 1	−2	−1	0
6.5	9.603	−1	.091	37	+14	+ 3	0	0	0
7.5	0.111	−1	.105	42	+17	+ 4	0	+1	+1
8.5	0.606	0	.127	43	+19	+ 4	+1	0	+1
9.5	1.079	0	.153	39	+15	− 2	0	+1	+1
10.5	1.526	+1	.184	32	+ 6	− 6	0	−1	0
11.5	1.942	+2	.218	22	− 1	−14	−2	−3	−2
12.5	2.324	0	.245	15	− 5	−15	+1	−2	+1
13.5	2.679	+2	.265	16	− 3	−12	+1	−3	+1
14.5	3.014	+3	−0.271	+32	+13	+ 5	+2	−9	−2
15.5	3.343	0	.....	.....	.....	.....	.....	.....	.....

Consider now the stars of magnitude m as expressed by equations (5). The contributions of the successive shells have an approximately Gaussian distribution. Hence the mean log ρ for all the stars of magnitude m equals the value of log ρ for maximum frequency. For the brighter stars the distances are such that the corresponding

[1] *Astronomische Nachrichten*, **213**, 47, 1921. These calculations were begun some years ago when no luminosity function for photographic magnitudes was available. It was not therefore convenient to use the Mount Wilson star counts. In the meantime van Rhijn's photographic luminosity function, *Groningen Publication* No. 38, has appeared; but, since the present example is only illustrative of the method, the older data are retained. Applications based on modern data will be made later.

[2] *Mt. Wilson Contr.*, No. 229, Table Ia; *Astrophysical Journal*, **55**, 245, 1922.

absolute magnitude is about -4.5, and we assume this value to represent maximum frequency for all values of m. The values of

TABLE II*a*

VALUES OF MEAN LOG ρ

m	(1)	(1*a*)	(2)
1.5	1.2	1.32	1.32
4.5	1.8	1.88	1.88
7.5	2.4	2.39	2.36
10.5	3.0	2.88	2.70
13.5	3.6	3.27	3.10
16.5	4.2	3.72	3.55

TABLE III

VALUES OF LOG d

LOG ρ	APPROXIMATION*				
	(1)	(2)	(3)	(4)	(5)
0.0	.....	−0.032	−0.031	−0.031	−0.031
0.2	.....	.037	.035	.035	.035
0.4	.....	.042	.039	.039	.039
0.6	.....	.047	.043	.043	.043
0.8	.....	.051	.047	.047	.047
1.0	−0.058	.056	.051	.051	.046
1.2	.061	.060	.055	.055	.042
1.4	.065	.065	.059	.059	.054
1.6	.069	.070	.062	.062	.072
1.8	.074	.075	.066	.066	.081
2.0	.082	.083	.071	.070	.085
2.2	.091	.099	.075	.074	.079
2.4	.105	.130	.084	.077	.059
2.6	.127	.185	.102	.083	.056
2.8	.153	.235	.198	.102	.080
3.0	.184	.265	.249	.253	.231
3.2	.218	.290	.271	.271	.346†
3.4	.245	.315	.292	.290	.461
3.6	.265	.340	.312	.308	.413
3.8	.271	.363	.334	.325	.325
4.0	.290	.386	.354	.345	− .222
4.2	.304	.410	.375	.364	+ .088
4.4	.317	.434	.398	.382	+ .256§
4.6	.330	.458	.418	.400	− .400
4.8	.344	.482	.438	.420	− .420
5.0	−0.358	−0.506	−0.458	−0.440	−0.440

* Values of log d for log $\rho < 1.0$ and > 3.8 are extrapolations. See Figures 1 and 2.
† Final value is −0.362. § Final value is +0.199.

mean log ρ corresponding to $m = 0.5, 1.5, \ldots$ are therefore 1.0, 1.2, ... (Table I) and are to be correlated, term by term, with the

data for log δ_m given in the fourth column of Table II. The results, which represent the first approximation for the relation between log ρ and log d, appear in the first two columns of Table III and as curve 1 in Figure 1. These values of log d give in turn a first approximation for D, which, by means of (5), may be used to improve the values of mean log ρ.

For use with (5) it is convenient to form at once the values of DV. Final choice of units will be made later; for the present we assume both D_9 and V_9 (shell having median log $\rho = 0.9$) to be unity. Equations (4) and (7) then give

$$\begin{aligned} &D_9 V_9 = 1 , \\ &D_{11} V_{11} = d_{10}\sigma , \\ &D_{13} V_{13} = d_{10}\sigma \cdot d_{12}\sigma , \\ &D_{15} V_{15} = d_{10}\sigma \cdot d_{12}\sigma \cdot d_{14}\sigma , \\ &\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned} \tag{10}$$

The successive values of log DV may therefore be found by adding log $\sigma = 0.6$ to each log d in the second column of Table III and forming the cumulative sums. Since these values of log d are the respective values of log δ_m, it happens that this first rough approximation for log DV is the original sequence of values for log $N(m)$ modified by a constant. Values of log DV for the final approximation, based on log d in the last column of Table III, are given in Table VI.

In using equations (5), Table I is the guide for combining DV with ϕ. Thus for $m = 1.5$ and log $\rho = 1.1$, $M = -4$. The value of $D_{11}V_{11}$ is therefore to be multiplied by $\phi\,(-4)$.

The revision of the values of mean log ρ does not require a separate calculation for each value of m; nor is it necessary to compute each term in the right member of (5). Since the relation between m and the corresponding mean log ρ is nearly linear, an interval of three or four magnitudes is sufficient. Further, since the distribution of n is approximately Gaussian, the values of log n define a parabola whose vertex coincides with maximum frequency. Hence if x_1, x_2, x_3 be three successive values of log n (preferably near the maximum),

and if y_2 be the value of log ρ corresponding to x_2, the vertex value of log ρ, for the intervals used here, is

$$y_0 = y_2 + \frac{1}{10}\,\frac{x_1 - x_3}{(x_1 - x_2) - (x_2 - x_3)}\,. \qquad (11)$$

Results thus found for six values of m are given in the third column of Table II*a*. These, together with interpolated data for in-

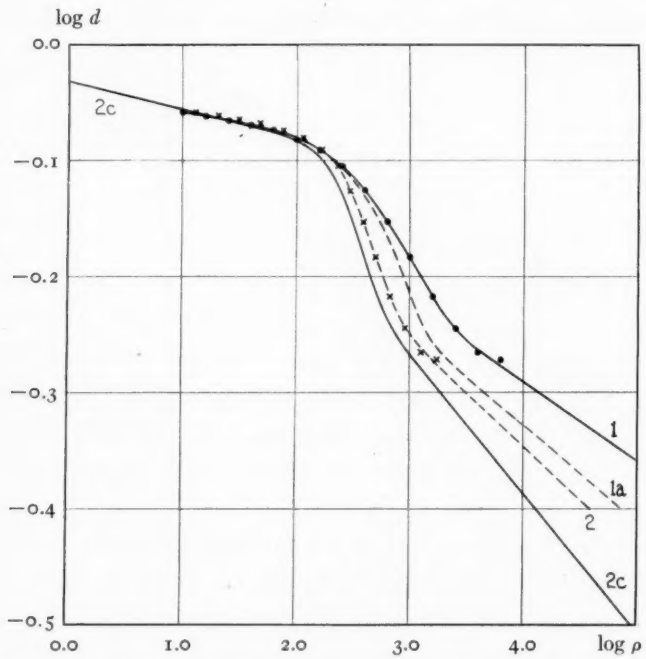

FIG. 1.—Preliminary approximations for log d

termediate values of m, are plotted against log δ_m as curve 1*a*, Figure 1. This curve is the basis for further improvement of log ρ, which leads finally to the last column of Table II*a* and to curve 2 of Figure 1. The values of log d in the third column of Table III were read directly from curve 2 for log $\rho = 0.0, 0.2, \ldots$ and represent the results of the second approximation.

The test of this approximation, as of all others, is afforded by a calculation of values of log δ_m by means of equations (10), (5), and

(9). When the results thus found agree with those derived directly from the star counts, the approximation for d, and hence for D, may be adopted as satisfactory. The manner of using (10) has already been described. In the case of (5), the calculation is to be extended over all shells which yield any appreciable contribution to stars of the magnitude in question. As before, Table I is the guide for combining ϕ and DV. The arrangement of the results is illustrated by Table VII, which gives the values of n for the final approximation.

The values of $\log \delta_m$ found from the second approximation for $\log d$ are plotted as curve *2c* in Figure 1. The corresponding residuals in the fifth column of Table II are all positive and show that for the third approximation the densities must be increased. The adopted corrections for $\log d$ are the differences in the ordinates of curves *2* and *2c;* the corrected values in the fourth column of Table III, in turn, lead to residuals (3) in Table II. A repetition of the process, with the relation between residuals (2) and (3) as a guide, gives the results found in Tables II and III under the heading (4).

The representation is now satisfactory to the sixth magnitude. The fourth approximation, however, is no real improvement on the third, and some other method of correction must be found which will bring to light the fluctuations obviously present in the values of $\log d$. Before this is attempted, a remark may be made concerning the revision of calculations involving equations (10) and (5), which may often be simplified by noting that the cumulative sums of the corrections to $\log d$ are directly the corrections to $\log DV$. Hence, corrections to n may be easily computed, and it is frequently advantageous to form these corrections instead of completely re-solving equations (5).

IV. APPROXIMATION BY STANDARD VARIATIONS

The residuals from the fourth approximation are to be reduced by further correction of $\log d$ to quantities comparable with the accumulated error of calculation,[1] say to two or three thousandths in $\log \delta_m$.

[1] This limit, which is far beyond the precision of the data, is chosen to illustrate the possibilities of the method. Since residuals (4) of Table II are no great improvement over those of the preceding column, the final approximation by the method given in this section might advantageously have been started after the third approximation.

Further, the final residuals must show an accidental alternation of sign. A direct solution is out of the question, first, because of the labor involved; and second, because inherent numerical uncertainty in the equations would transform small residual errors in the data into inadmissible fluctuations in the density function. Various approximation methods were tried, none of which would converge. Finally a procedure was devised which proved successful and much more expeditious than might be supposed.

In solutions based on the ordinary method of differential correction, we form the total differential of the function for simultaneous changes in all of the variables. In what follows we use a series of partial differentials of the function, each corresponding to a standard differential change in a single variable. From these we build up a series of changes in the variables, which, made simultaneously, produce a series of total differential changes in the function equal to the given series of residuals. Thus, for the present problem we make a standard change of ± 0.05 in $\log d$, in succession for each value of $\log \rho$, and calculate in each instance the corresponding changes in $\log \delta_m$. The required solution is then found by combining these standard residuals, multiplied by appropriately chosen factors, in such a way as to reproduce the original residuals.

Let

$$d' = (1+a)d \,, \tag{12}$$

where d and d' represent original and changed values of the relative density. Equations (5) and (10) then give for the corrected number of stars of magnitude m

$$N'(m) = N(m) + a\sum_s n(m) \,,$$

where the summation begins with the shell whose inner surface marks the point at which d has been changed. Similarly, for magnitude $m+1$ the same change, made at the same point, gives

$$N'(m+1) = N(m+1) + a\sum_s n(m+1) \,.$$

Hence, by (9), the corrected δ_m is

$$\delta'_m = \delta_m \frac{1+R_s^{m+1}}{1+R_s^m}, \tag{13}$$

where

$$R_s^{m+1} = \frac{a}{N(m+1)} \sum_s n(m+1); \qquad R_s^m = \frac{a}{N(m)} \sum_s n(m). \tag{14}$$

For the standard change

$$\pm \Delta \log d_s = \log(1 \pm a), \tag{15}$$

we have therefore the standard residuals

$$\pm \Delta \log \delta_m = \log (1 \pm R^{m+1}) - \log(1 \pm R_s^m), \tag{16}$$

where the index m is to be extended over all magnitudes which yield appreciable values for the residuals.

If the standard positive and negative changes in log d are numerically equal, the respective values of a will not be exactly the same. It is simpler, however, to use a constant a and allow the positive and negative changes to differ. Thus for the change -0.050, $a = -0.109$; for $a = +0.109$, the corresponding standard positive change is $+0.045$.

To apply these formulae we begin with the table for n derived with the fourth approximation (the equivalent of Table VII) and form for each column the cumulative sums from the bottom upward. Multiplication of these sums for column m by $a/N(m)$ gives the values of R_s^m, where s changes from line to line of the table. For example, values for the line log $\rho = 2.1$ refer to the change made at the inner boundary of the corresponding shell, that is, at log $\rho = 2.0$, for which $s = 20$. The remainder of the calculation by equation (16) needs no comment.

Table IV illustrates a portion of the table of residuals (unit = 0.0001) corresponding to changes of $+0.045$. Thus if log d for log $\rho = 2.4$ be increased by this amount and no other change be made, the computed values of log δ_m will be increased by the quantities given in the first line of Table IV, while residuals (4) of Table II will be diminished by the same amounts.

The use of these data to build up a solution which will represent the star counts depends on the fact that moderate variations in the standard changes produce approximately proportional variations in the standard residuals. Thus if P times a certain series of

TABLE IV

STANDARD RESIDUALS: VALUES OF $\Delta \log \delta_m$ CORRESPONDING TO A CHANGE OF +0.045 IN LOG d AT THE DISTANCE LOG ρ (UNIT=0.0001)

Log ρ	m														
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5
2.4....	4	10	21	32	47	61	65	60	49	37	26	17	8	6	1
2.6....	....	5	10	16	30	45	55	64	55	50	41	31	20	13	6
2.8....	....	1	5	6	14	26	37	48	54	57	53	47	35	27	18
3.0....	....	....	1	2	3	11	19	27	37	47	53	56	53	46	35
3.2....	....	....	....	....	1	3	8	12	20	30	41	50	57	58	52

standard residuals is used, a must be replaced by fa, where f is defined by

$$P \log \left(\frac{1 \pm R^{m+1}}{1 \pm R^m} \right) = \log \left(\frac{1 \pm fR^{m+1}}{1 \pm fR^m} \right) . \tag{17}$$

The corresponding factor Q for the standard change in log d is then given by

$$Q \log (1 \pm a) = \log (1 \pm fa) . \tag{18}$$

Expansion of the logarithms gives the approximate relations

$$\left.\begin{aligned} P(\pm 1 - \tfrac{1}{2}aq) &= \pm f(1 \mp \tfrac{1}{2}afq) \\ Q(\pm 1 - \tfrac{1}{2}a) &= \pm f(1 \mp \tfrac{1}{2}af) \end{aligned}\right\} , \tag{19}$$

where

$$q = \frac{\sum n(m+1)}{N(m+1)} + \frac{\sum n(m)}{N(m)} , \qquad (0 \ll q \ll 2) . \tag{20}$$

These relations show that in general Q differs from P by a quantity of the order of a and that for values equal to unity or less the difference between P and Q may be ignored. For cases in which large factors are required and for which the proportionality of

changes does not hold, the standard residuals may still be used by a simple modification of the procedure which will be illustrated later.

The problem is now to find by trial a group of factors such that when multiplied into the respective series of standard residuals, positive and negative, the sum of the modified series reproduces residuals (4) of Table II. The start is made by noting that a positive change is required near $m = 7.5$, and a negative change for $m = 9.5$–13.5. Reference to Table IV shows that the maximum effect for

TABLE V

FACTORS AND CORRESPONDING CORRECTIONS TO LOG d

(Fifth Approximation)

Log ρ	Factor	Δ log d	Log ρ	Factor	Δ log d
1.0	+0.1	+0.004	2.8	+ 0.5	+0.022
1.2	+0.3	+ .013	3.0	+ 0.5	+ .022
1.4	+0.1	+ .004	3.2	− 1.5	− .075
1.6	−0.2	− .010	3.4	− 3.2	− .171
1.8	−0.3	− .015	3.6	− 2.0	− .105
2.0	−0.3	− .015	3.8	0.0	.000
2.2	−0.1	− .005	4.0	+ 3.0	+ .123
2.4	+0.4	+ .018	4.2	+16.0	+ .452
2.6	+0.6	+0.027	4.4	+30.0	+0.638

these magnitudes is produced by changing log d near log $\rho = 2.6$ and 3.2, respectively. In choosing factors, allowance must be made for the fact that overlapping series with opposite signs partially neutralize each other; further, the curve of corrections must be made as smooth as possible by distributing large changes over several adjacent values of log d. The criterion of continuity in log d is a helpful guide, and a diagram like Figure 2 facilitates the choice of the factors. Those used in the present case are given in the second column of Table V. The corresponding changes in log d in the following column were mostly obtained by multiplying the positive and negative factors by the standard changes +0.045 and −0.050, respectively. For the cases involving large factors, values of f were computed by (17), which were then used with (18) to obtain the corrections to log d. The corrected values of log d are under heading (5) in Table III.

The final residuals to be expected as a result of these changes are given by

$$\text{Residual } (5a) = \text{Residual } (4) - \Sigma P \times \text{Standard Residual.} \qquad (21)$$

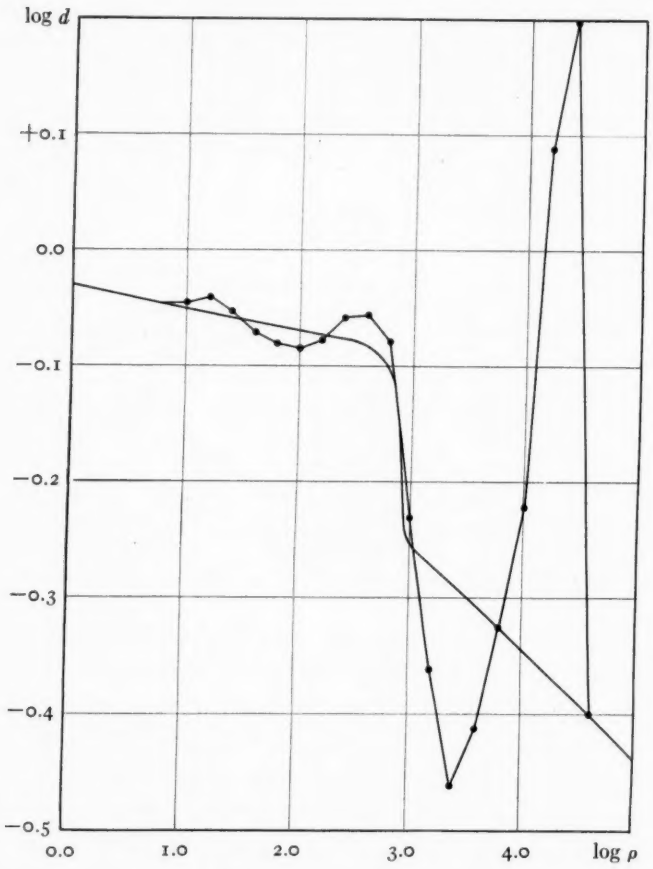

FIG. 2.—Final approximation for log d

The results, which appear under heading (5a) in Table II, are all small and show a proper alternation of sign.

The real test, however, is the representation of the observed values of log δ_m afforded by equations (5) and (9). The residuals from this test, (5), Table II, are essentially the same as (5a), as they should be, except for the last three values. The disagreement here arises

from equation (21), which assumes the relation between P and Q to be constant, whereas equations (19) and (20) show that it varies with m. When the factors are large, the resulting error is not inappreciable. An inspection of the calculation suggests the adjustments at log $\rho = 3.2$ and 4.4 given in the footnote to Table III. The

TABLE VI

FINAL DENSITIES

Median Log ρ	Log DV	Log D	Log D_0	$S-K$ and vR
−0.1	−2.805	+0.195	−0.781	+0.57
+0.1	2.236	0.164	0.812	+ .54
0.3	1.671	0.129	0.847	+ .50
0.5	1.110	0.090	0.886	+ .46
0.7	−0.553	+0.047	0.929	+ .43
0.9	0.000	0.000	0.976	+ .37
1.1	+0.554	−0.046	1.022	+ .33
1.3	1.112	0.088	1.064	+ .29
1.5	1.657	0.142	1.118	+ .23
1.7	2.185	0.214	1.190	+ .16
1.9	2.704	0.295	1.271	+ .08
2.1	3.219	0.380	1.356	− .01
2.3	3.740	0.459	1.435	− .08
2.5	4.281	0.518	1.494	− .09
2.7	4.825	0.574	1.550	− .04
2.9	5.346	0.654	1.630	+ .05
3.1	5.715	0.885	1.861	+ .04
3.3	5.953	1.248	2.224	− .06
3.5	6.092	1.709	2.685	− .21
3.7	6.279	2.122	3.098	− .26
3.9	6.554	2.447	3.423	− .20
4.1	6.932	2.669	3.645	+ .06
4.3	7.620	2.581	3.557	+0.63
4.5	8.419	2.382	3.358	+1.36
4.7	8.619	2.782	3.758	+1.59
4.9	8.799	3.202	4.178	+1.42
+5.1	+8.959	−3.642	−4.618	+2.08

corresponding values of log DV are in Table VI; n and $N(m)$, in Table VII; and the final residuals for log δ_m in the last column of Table II.

An even better test is the representation of the original star counts in the second column of Table II. These counts refer to an area of 1 sq. deg., but since V represents the volume of a complete shell, the values of $N(m)$ in Table VII (unit indicated in last line) apply to the whole sky. Further, the units for both D and V have arbitrarily been assumed to be the density and the volume of the

shell for which median log $\rho = 0.9$. To make the results comparable, the observed counts may also be referred to the whole sky, while the values of $N(m)$ in Table VII are multiplied by a factor c which transforms them into actual numbers of stars. The comparison itself gives log $c = 2.522$, and finally the residuals in log $N(m)$ shown in the

TABLE VII

RELATIVE NUMBERS OF STARS OF MAGNITUDE m AT DIFFERENT DISTANCES FROM THE SUN

MEDIAN LOG ρ	MEDIAN m															
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5
−0.1	20	2	2													
+0.1	52	7	9	1												
0.3	115	19	27	3	3											
0.5	214	42	69	10	12	1	1									
0.7	340	77	151	25	35	4	4									
0.9	454	121	276	54	89	13	15	2	1							
1.1	518	163	434	99	193	32	45	5	6							
1.3	509	187	588	157	357	70	116	16	20	2						
1.5	415	179	656	206	551	125	244	41	58	7	7	1				
1.7	277	140	603	221	695	186	423	82	137	19	23	2	2			
1.9	154	92	462	199	731	230	614	140	272	45	64	8	8	7		
2.1	73	51	300	151	652	239	752	201	457	89	148	21	25	26	2	2
2.3	30	24	168	100	502	216	794	250	667	152	296	49	70	84	9	8
2.5	10	10	84	58	346	175	752	276	867	232	527	103	171	242	29	30
2.7	3	4	36	29	204	121	611	263	966	303	811	184	360	597	85	102
2.9	....	1	13	12	77	68	402	203	873	321	1007	269	612	1194	198	281
3.1	....	....	4	3	28	23	158	94	474	204	750	236	630	1432	279	463
3.3	....	....	....	....	5	5	39	27	163	82	353	130	407	1089	248	483
3.5	....	....	....	....	....	1	7	5	38	22	113	49	179	561	150	341
3.7	....	....	....	....	....	....	1	1	8	6	34	17	75	275	86	231
3.9	....	....	....	....	....	....	....	....	2	2	11	6	33	141	52	163
4.1	....	....	....	....	....	....	....	....	....	....	4	3	15	78	34	124
4.3	....	....	....	....	....	....	....	....	....	....	2	2	13	76	38	164
4.5	....	....	....	....	....	....	....	....	....	....	....	1	12	80	48	240
4.7	....	....	....	....	....	....	....	....	....	....	....	....	2	18	13	75
4.9	....	....	....	....	....	....	....	....	....	....	....	....	....	3	3	19
+5.1	....	....	....	....	....	....	....	....	....	....	....	....	....	1	0	4
$N(m)$	3191	1119	3882	1328	4480	1509	4978	1606	5009	1486	4150	1081	2614	5904	1274	2730
Unit	10^{-5}	10^{-4}	10^{-4}	10^{-3}	10^{-3}	10^{-2}	10^{-2}	10^{-1}	10^{-1}	1	1	10	10	10	10^{2}	10^{2}

third column of Table II. The small systematic trend in differences reflects the slight tendency toward clustering shown by the positive values in the last column of the table.

V. FINAL DENSITIES—NUMBER OF STARS PER CUBIC PARSEC

The cumulative sums of the final values of log d in Table III are the adopted values of log D (Table VI), for which, as heretofore, $D = 1$ for the shell having median log $\rho = 0.9$. The density expressed in stars per cubic parsec within any shell may be found from the

corresponding line of Table VII. For any such line the horizontal sum is

$$S=\sum_{m_1}^{m_2} n(m)=DVs\,, \qquad s=\sum_{M_1}^{M_2}\phi(M)\,, \tag{22}$$

where m_1 and m_2 are the apparent magnitudes within which n has appreciable values and M_1 and M_2 the corresponding absolute magnitudes. To obtain the total number of stars in the shell, S must be multiplied by the constant c of the preceding section; further, V is to be expressed in cubic parsecs. The required density then becomes

$$D_0=D\Delta_0=\frac{cS}{sV}\,, \tag{23}$$

where Δ_0 is the number of stars per cubic parsec in the shell for which D is unity. The constant Δ_0 may be computed from the data for any shell, or, as a check, from two or more shells. Thus for $\log\rho=$ 0.9, 1.9, and 2.9, $\log\Delta_0=9.026$, 9.024, and 9.025, respectively.

The corresponding values of $\log D_0$ in the fourth column of Table VI may be compared with the results of Kapteyn and van Rhijn ($\log\Delta$).[1] From about 100 parsecs outward their densities were derived from exactly the same data as used here, on the assumption that Δ has the form of a quadratic exponential in $\log\rho$. For the interval 100–2000 parsecs, the values of $\log D_0/\Delta$ in the fifth column of Table VI correspond to an average difference of about 12 per cent. At greater distances the difference in the two solutions is much larger. The uncertainty in the data is such, however, that probably there is little choice between the two series of results; and the point made is merely that the present solution closely represents the initial data. Near the sun, $\log\Delta$ depends on different data, treated by a different method. The practical constancy of Δ in this region is confirmed by a later investigation by van Rhijn[2] and cannot be seriously questioned. The significance of the discordant values of

[1] *Mt. Wilson Contr.*, No. 229, Table Ib, second column; *Astrophysical Journal*, **55**, 246, 1922.

[2] *Groningen Publication*, No. 38, p. 15, 1925.

log D_0 in the vicinity of the sun is discussed in the following *Contribution;* here it need only be noted that D_0 represents the star counts.

VI. FINAL REMARKS

In conclusion, it may be noted again that the method imposes no restriction on the form of the distribution functions for apparent and absolute magnitudes. Further, allowance for any dependence of the luminosity function on distance is easily made by using in equations (5) the frequencies for M which are appropriate to the respective shells. The illustrative example has been treated with a constant ϕ because hitherto we have had no reason for doing otherwise. The rapid change in the resulting densities in the neighborhood of the sun indicates, however, that this function is not everywhere the same. See *Contribution* No. 437.

The necessity for extrapolating the curves for log d suggests a certain arbitrariness in the solution. The results do depend on the extrapolation, but the analytical methods likewise involve an extrapolation in that the function which represents the star counts is tacitly assumed to represent fainter stars not included in the counts. Table VII illustrates the difficulty. Magnitudes 12–16, for example, include a certain percentage of stars which are more than 10,000 parsecs distant. The distribution of these stars depends on the density at great distances, which for its precise determination would require counts of very faint stars. Since these counts are not available, extrapolations of counts of the brighter stars must be used. Table VII indicates, however, that up to 10,000 parsecs the uncertainty arising from the extrapolation is not very serious. The total number of stars in any column is determined by observation; the percentage beyond 10,000 parsecs is not large. Hence some uncertainty in this percentage does not greatly affect the distribution of the nearer stars of the same apparent magnitude.

Although the initial star counts are well represented, the calculated densities are not strictly correct because no allowance has been made for certain small statistical errors which enter at various points in the calculation. For example, the values of M in Table I correspond to the median values of log ρ and of m within certain intervals and do not exactly equal the mean values of M within these

same intervals. Again, the mean density of a shell does not occur at a distance corresponding to median log ρ. Correction is easily made whenever it is worth while to do so.

Finally, it may be noted that statistical relations between apparent and absolute magnitudes, distance, and parallax—for example, the mean parallaxes of stars of magnitude m—may easily be formed from the array of numbers in Table VII.

I am greatly indebted to Miss Joyner of the Observatory staff for much assistance with the computations required for the foregoing discussion.

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August 1931

REVIEWS

Wavelength Tables for Spectrum Analysis. Second edition. By F. TWYMAN and D. M. SMITH. London: Adam Hilger, Ltd., 1931. Pp. xii+180. 14*s*. 9*d*., post free.

Those who have used the first edition of the *Wavelength Tables* will find in this edition a much improved book. Besides a revision of the wave-lengths which were used merely for the identification of an element, the authors have added information regarding, and methods of making, quantitative analyses with the aid of spectroscopy. As an introduction for the practical worker to the theory underlying modern methods a section on "The Various Types of Spectrum" by Professor E. N. da C. Andrade has been inserted. The new edition considers analysis by the arc and flame methods and has a table of the "Sensitive Arc Lines of 50 Elements" which lists the lines according to wave-length for each element, gives the intensities and the relative sensitivities of the lines. Another section deals with the characteristics and use in analysis of the sensitive lines in the arc and spark spectra. This section contains a classification of the ultimate lines of 61 elements, the table containing the wave-length, the intensities in arc and spark, the series relations, the quantum number Δk, the authorities, the excitation potentials, and the temperature classification of A. S. King.

Part IV of the first edition, "Wavelengths for Use in Determination of Stellar Radial Velocities," has been omitted. This is quite justified, since the book was prepared for the use of chemists and metallurgists in spectrum analysis. The astronomers have access to the tables in other publications; and besides, it is an opportune time for a book to be written on stellar spectra, collecting together a revision of tables of wave-lengths suitable for the determination of radial velocities, the identifications of lines, and a discussion of the variations of the lines with the physical conditions in the stellar atmosphere.

C. T. ELVEY